



Specifications  
of  
RealVol Indices

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## Table of Contents

Overview .....	4
Volatility Defined .....	4
Measurement, Not Observation .....	4
RealVol Indices .....	4
Goal .....	5
RealVol Daily Index Types and Time Frames .....	5
RealVol Underlying Assets .....	5
RealVol Symbols .....	5
Exhibit 1: Table of Symbols .....	7
Exhibit 2: Table of Underlying Assets .....	8
Daily Versus Real-Time .....	10
Design of RealVol Real-Time Formula .....	10
RealVol Daily Indices (Details) .....	11
Terms .....	13
Realized Volatility .....	13
Implied Volatility .....	13
Underlying .....	13
Underlying Reference Price (URP) .....	13
Defined Days .....	14
Formulas .....	15
Design of RealVol Daily Formula .....	15
RealVol Daily Formula .....	15
RealVol Real-Time Formula .....	16
RealVol Overnight/Intraday Formula .....	16
RealVol Correlation Formula .....	17
RealVol Variance Formula .....	17
RealVol Rough Vol Formula .....	17
RealVol HARK Vol Formula .....	17
Formula Special Considerations .....	19
Time of Day .....	19
Non-Trading Days .....	19
Exhibit 3 .....	21
Notes .....	21
Phantom Volatility .....	23
Stocks Paying a Dividend .....	23
Stock Splits, Reverse Splits, Stock Dividends .....	23
Expiring Underlyings .....	23
Adjusting for Phantom Volatility .....	25
Adjustment for Dividends .....	25
Dividends .....	25
Adjustment for Stock Splits .....	25
Adjustment for Expiring Underlyings .....	26
Process for Market Disruption Events .....	27
Surrogate Markets .....	27

Using Returns versus URPs.....	27
Complete Closure of the Primary Market .....	27
Partial Closure of the Primary Market (but Closing Normally) .....	28
Partial Closure of the Primary Market (and not Closing Normally) .....	28
Calculation Examples of VOL Daily Indices .....	29
VOLm Index.....	29
VOLq Index.....	30
VOLy Index.....	31
Exhibit 4.....	31
Shorthand .....	32
Exhibit 5.....	33
Notice .....	34

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## Overview

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### Volatility Defined

In the financial markets, all investments have risk. Because of risk, there also exists opportunity. Many people automatically think of risk as “bad,” or synonymous with *risk of loss*. However, the true definition of risk encompasses not only the potential for loss, but the potential for gain as well. One of the ways to measure price risk is known as “volatility.” This is why volatility is so important in the endeavor of investing and risk management. In its simplest description, volatility is “magnitude of movement regardless of direction.” In other words, volatility deals only with price change, but is indifferent as to whether that change is up or down.

### Measurement, Not Observation

One of the key issues regarding realized volatility is that it is *measured* not *observed*. In other words, one cannot look at a particular price and determine what volatility that represents. The only way to calculate realized volatility is to get prices over time and then use a realized volatility formula to calculate the movement of prices for that time frame. This is a key concept because all realized volatility formulas must use historical prices in order to calculate a result.

For example, if the evening news reports that the price of gold is currently \$1,000 per ounce, it has expressed only the *price* of gold. The current price says nothing about how much gold had to move to get to \$1,000. Did it just move down from \$2,000? Or, did it just move up from \$999? One cannot tell unless previous (historical) data are also used. Thus, saying that the price of gold is now \$1,000 tells us nothing about its volatility, because we lack a historical perspective.

### RealVol Indices

This document describes the process of creating RealVol Indices to measure realized volatility and related statistical methods in a standardized manner on a daily and real-time basis. The one real-time version is trademarked under the name *RealVol Real-Time Index*. The other versions will be updated once per day (“daily”) and trademarked under the name *RealVol Daily Indices*. The flagship set of RealVol Indices attempt to measure the movement, regardless of direction, of some underlying, over a predefined time frame.

## Goal

Every index that we can think of *combines* assets together into one. For example, equity indices typically combine many, stocks into one index. A bond index typically combines a plethora of individual bonds into one index. In contrast, RealVol indices *break apart* a single asset's risks into its components. Because risk is notoriously difficult to standardize, the team at Demand Derivatives devised a methodology that standardizes risk components. They have done so by breaking down risk into various types and several time frames, which are designed to match the preferred risk measure and time horizon of the vast majority of traders.

## RealVol Daily Index Types and Time Frames

There are 40 RealVol Daily Indices: seven types in each of six time frames (see Exhibit 1). Note: Only 40 of the potential 42 indices are calculated (1-day and 1-week correlation are not). Also, the abbreviation "vol" means realized volatility.

### Seven Types

1. Realized vol ("VOL")
2. Realized vol of vol ("VOV")
3. Overnight/intraday vol ("DVOL")
4. Correlation (underlying vs. vol) ("VCOR")
5. Realized variance ("VAR")
6. Rough forecast vol ("RVOL")
7. HARK forecast vol ("HVOL")

### Six Time Frames

1. 1 day ("d")
2. 1 week (5 trading days) ("w")
3. 1 month (21 trading days) ("m")
4. 1 quarter (63 trading days) ("q")
5. 1 half year (126 trading days) ("h")
6. 1 year (252 trading days) ("y")

## RealVol Underlying Assets

All 40 index types are furnished for each underlying asset. In this case, there are 40 underlying assets (see Exhibit 2) — 1,600 in all (40 types x 40 underlying assets). The 40 assets are all futures contracts except for the SPDR S&P 500® ETF (symbol SPY). These contracts have some of the highest volumes in the world and are widely followed by the industry and other market participants.

## RealVol Symbols

Combining the symbology above for the type and the time frame yields a generic symbol such as VOLd (1-day realized volatility),

VCORm (21-day correlation), HVOLy (252-day HARK forecast volatility), etc. Finally, adding the underlying asset to the mix creates a complete symbol such as VOVwC (5-day volatility of volatility on Corn), VARqGC (63-day variance on gold), RVOLhTY (126-day Rough forecast volatility on 10-year Treasury Notes), etc.

Exhibit 1: Table of Symbols

Base Symbol	Description	Day (1-day)	Week (5-day)	Month (21-day)	Quarter (63-day)	Half Year (126-day)	Year (252-day)
VOL	Realized Volatility	VOLd	VOLw	<u>VOLm</u>	VOLq	VOLh	VOLy
VOV	Realized Volatility of Realized Volatility	VOVd	VOVw	VOVm	VOVq	VOVh	VOVy
DVOL	Overnight/Intraday "Daily" Realized Volatility	DVOLd	DVOLw	DVOLm	DVOLq	DVOLh	DVOLy
VCOR	Correlation of Underlying vs. VOL	N/A	N/A	VCORm	VCORq	VCORh	VCORy
VAR	Realized Variance	VARd	VARw	VARm	VARq	VARh	VARy
RVOL	RFSV "Rough" Model Forecast of VOL	RVOLd	RVOLw	RVOLm	RVOLq	RVOLh	RVOLy
HVOL	HARK Model Forecast of VOL	HVOLd	HVOLw	HVOLm	HVOLq	HVOLh	HVOLy

Exhibit 2: Table of Underlying Assets

Asset Class Symbol	Index Name	Underlying Asset	Start
<b>Equities</b>			
VOLSPY	RealVol SPY Indices	SPDR® S&P 500® ETF	1993
<b>Equity Indices</b>			
VOLES	RealVol US 500 Indices	E-mini® S&P 500® Futures – CME	1997
VOLYM	RealVol Indu 30 Indices	E-mini Dow® (\$5) Futures – CBT	1997
VOLNQ	RealVol Tech 100 Indices	E-mini NASDAQ 100® Futures – CME	1999
VOLMD	RealVol Mid 400 Indices	E-mini S&P® MidCap400® Futures – CME	1992
VOLNK	RealVol Japan 225 Indices	Nikkei®/USD Futures – CME	1990
<b>Commodities</b>			
VOLCC	RealVol Cocoa Indices	Cocoa Futures – ICE	1970
VOLKC	RealVol Coffee Indices	Coffee Futures – ICE	1973
VOLC	RealVol Corn Indices	Corn Futures – CBT	1962
VOLCT	RealVol Cotton Indices	Cotton Futures – ICE	1972
VOLFC	RealVol Feeder Cattle Indices	Feeder Cattle Futures – CME	1974
VOLLC	RealVol Live Cattle Indices	Live Cattle Futures – CME	1966
VOLLN	RealVol Lean Hogs Indices	Lean Hog Futures – CME	1970
VOLOJ	RealVol Orange Juice Indices	Orange Juice Futures – ICE	1967
VOLS	RealVol Soybean Indices	Soybean Futures – CBT	1970
VOLSBM	RealVol Soybean Meal Indices	Soybean Meal Futures – CBT	1964
VOLBO	RealVol Soybean Oil Indices	Soybean Oil Futures – CBT	1962
VOLSB	RealVol Sugar Indices	Sugar Futures – ICE	1964
VOLWHT	RealVol Wheat Indices	Wheat Futures – CBT	1962
<b>Currencies</b>			
VOLSF	RealVol CHF/USD Indices	Swiss Franc Futures – CME	1975
VOLEC	RealVol EUR/USD Indices	Euro FX Futures – CME	1999
VOLBP	RealVol GBP/USD Indices	British Pound Futures – CME	1976
VOLJY	RealVol USD/JPY Indices	Japanese Yen Futures – CME	1977
<b>Interest Rates</b>			
VOLTU	RealVol 2 Yr Note Indices	2 Yr Note Futures – CBT	1990
VOLFV	RealVol 5 Yr Note Indices	5 Yr Note Futures – CBT	1988
VOLTY	RealVol 10 Yr Note Indices	10 Yr Note Futures – CBT	1982
VOLUS	RealVol U.S. T-Bond Indices	U.S. Treasury Bond Futures – CBT	1977
VOLBS	RealVol Euro-Schatz Indices	Euro-Schatz – EUREX	1997
VOLBM	RealVol Euro-Bobl Indices	Euro-Bobl – EUREX	1995
VOLBL	RealVol Euro-Bund Indices	Euro-Bund – EUREX	1990
VOLLG	RealVol Long Gilt Indices	Long Gilt – LIFFE	1990

**Energy**

VOLBC	RealVol Brent Crude Oil Indices	Brent Crude Oil Futures – ICE	1994
VOLCL	RealVol Crude Oil Indices	Crude Oil Futures – NYMEX	1983
VOLG	RealVol Gasoil Indices	Gasoil Futures – ICE	1990
VOLHO	RealVol Heating Oil Indices	NY Harbor ULSD Futures – NYMEX	1980
VOLNG	RealVol Natural Gas Indices	Nat Gas (H.H.) Physical Futures – NYMEX	1990
VOLRB	RealVol NYH RBOB Gas Indices	RBOB Gasoline Physical Futures – NYMEX	1996

**Metals**

VOLHG	RealVol Copper Indices	Copper Futures – CEC	1962
VOLGC	RealVol Gold Indices	Gold Futures – CEC	1975
VOLSLV	RealVol Silver Indices	Silver Futures – CEC	1965

## Daily Versus Real-Time

No index, that Demand Derivatives is aware of, currently has both daily and real-time versions simultaneously. Most indices are updated on a real-time, or nearly real-time, basis. Some are calculated daily. But, none have both. Why the distinction in this case? Typically, volatility is measured on a daily basis only. Therefore, Demand Derivatives created the RealVol Daily Indices to correspond to the standard of using only daily (i.e., closing) underlying reference prices ("URPs"). The VOL Index price on any particular day is the value that will be used to settle tradable RealVol Instruments.

However, traders often demand indices that are updated more frequently. The problem is how to furnish a real-time version for a daily volatility index. Demand Derivatives solved this conundrum by taking today's URP before the close (numbered day #22) and weighting it by the proportion of time through today's trading day, then using the remaining weight for the first URP (day #1). In essence, the first day of the period (22 trading days prior, day #1) and the last day of the period (today, day #22) will have a combined weight of 100% in total, while the days in between will have a weight of 100% each. In this way, the 22 returns will be weighted as if there are only 21 returns and the RealVol Index will, therefore, be updating throughout the day as today's URP changes. Note: the RealVol Real-Time Formula results in exactly the same value as the RealVol Daily Formula at the instant when the market closes.

## Design of RealVol Real-Time Formula

All of the design elements described for the RealVol Daily Formula are the same for the RealVol Real-Time Formula. To convert from a daily to a real-time value, one needs to start with the RealVol Daily Formula, then incorporate the current underlying price and a weighting scheme. Doing so provides continuous updates throughout the trading day and delivers to traders a useful, real-time indication of the up-to-the-moment 21-day daily realized volatility. At Demand Derivatives, it was decided to convert only the VOLm Index to a real-time index (VOL). Essentially, VOL measures a constant 21-day realized volatility even while we are within the new, most recent, day ("Today").

For instance, if we are 80% through the current day (n+1), we will use the most up-to-the-moment URP to calculate the partial (80%) day's return (n+1) from yesterday's URP (n). Then we consider the very first day in the calculation period and weight that whole day's return by 20% (100% - 80% = 20%). In this manner, we still have the weight of 21-day realized volatility at any moment in time even though there are actually 22 returns — 20% weight on day 1, 80% weight on day 22,

and full weights for days 2 through 21 (for a total weight of 21 days of returns).

Note: While the partial return of the current day is self-weighting, and therefore requires no additional coefficient, the self-weighted portion of the current day is nonetheless required to be calculated so as to apply the proper remaining weight to the full day 1 return. In order to calculate the weight of the current day, the current time each day is measured to the closest second. Since there are 86,400 seconds in a day, the current time and the number of seconds in a day are used in the RealVol Real-Time Formula to calculate the daily weight to be applied to day 1.

When the time of day equals the close of today (n+1), the weight of the return of day n+1 is now 100%, while the weight of the return of day 1 is 0%. Thus, with its weight of zero, the return of the original day 1 drops out of the calculation. The original day 2 now becomes the new day 1 and all other days get renumbered as well. The RealVol Real-Time Formula at this very instant in time (the close at 4:00 PM ET in our example) simplifies to the RealVol Daily Formula. The instant after the market closes, we begin anew, with the returns renumbered, such that there are again only 21 returns, with the new trading day having a weighted return as day 22.

Note that the new "day" begins immediately after the close of the market on a particular day (ignoring holidays and weekends), not necessarily at the end of the day (i.e., midnight).

## RealVol Daily Indices (Details)

### VOL Indices

Realized volatility indices (VOL) measure the interday (close-to-close) realized volatility of an underlying using the RealVol Daily Formula. One can think of VOL as a measure of interday price risk of an underlying. It is especially useful as a guide for traders of options in the underlying.

### VOV Indices

Realized volatility of realized volatility indices (VOV) measure the realized volatility of VOL using the RealVol Daily Formula a second time on the same data. Trading instruments on realized volatility compels one to discover the risk of those instruments. To do so, one needs to calculate the vol of vol. The VOV indices attempt to quantify such risk and can be especially useful as a guide to options traders of realized volatility instruments.

### DVOL Indices

Overnight/intraday realized volatility indices (DVOL) measure the overnight/intraday realized volatility from previous day's close and

today's open, high, and low data of an underlying using the RealVol Overnight/Intraday Formula. One can think of DVOL as a measure of overnight/intraday price risk of an underlying. It can be especially useful as a guide to timing trades in the underlying.

#### **VCOR Indices**

Correlation indices (VCOR) measure the correlation between the underlying and its VOL. Use VCOR for insight into the relationship between the underlying and its realized volatility.

#### **VAR Indices**

Realized variance indices (VAR) measure the interday (close-to-close) realized variance of an underlying using the RealVar Daily Formula. One can think of VAR as another measure of interday price risk of an underlying. Variances are easier to sum, average, and combine because the result is linear as opposed to volatility, which is a curve function.

#### **RVOL Indices**

The Rough Vol model (Rough Fractional Stochastic Volatility or "RFSV") forecasts realized volatility. According to the model, created by Professor James Gatheral of Baruch College, the log of daily high/low realized volatilities is well approximated by fractional Brownian motion with a Hurst parameter  $H$  close to zero. The Rough Vol model is used to create the RVOL indices, which forecast realized volatility over six standardized time frames.

#### **HVOL Indices**

HARK (Heterogeneous Auto-Regressive model cast into a Kalman filter framework) is a forecast of realized volatility created by Professor Fulvio Corsi of the University of Pisa and City University of London. It is a dynamic extension of the asymmetric (i.e., with leverage effects) HAR model where the parameters are continuously and optimally updated by the Kalman filter according to the statistical properties of an intraday realized volatility input. This allows flexibility and fast adaptation to the original HAR model. The HARK model is used to create the HVOL indices, which forecast realized volatility over six standardized time frames.

### Realized Volatility

Realized volatility is the “actual volatility,” “statistical volatility,” or “asset volatility” that the underlying has displayed over a specific period. The term “realized volatility” is very closely related to “standard deviation.” Realized volatility is a specific form of standard deviation. If one were to use daily returns of an underlying (instead of actual prices) and annualize the results, standard deviation becomes realized volatility. Demand Derivatives uses a modified version of the standard deviation formula. We will refer to the RealVol version of realized volatility as “realized volatility,” “realized vol,” or simply “vol.”

### Implied Volatility

There is another type of volatility known as “*implied volatility*.” Implied volatility is based on the relative expensiveness of associated options premiums. Implied volatility is a completely different approach to expressing volatility and often differs in value from realized volatility. The intricacies of implied volatility are beyond the scope of this document; hence, when we use the term “volatility,” we are referring to realized volatility.

### Underlying

The “underlying” can be just about anything that has a daily price. For example, the underlying can be a physical asset, security, futures contract, index, currency, bond, swap, measurement, etc.

### Underlying Reference Price (URP)

We will use the term Underlying Reference Price (URP) to refer to the actual daily closing price that the underlying has displayed, or will go on to display. The URP is the “closing,” “final,” or “settlement” price for the day. The URP is an especially attractive value for calculating Realized Volatility because of its ease of use, transparency, and wide dissemination to market participants. Roughly speaking, therefore, the underlying is the “asset,” while the URP is the “closing price.”

However, there are two exceptions: The first is when there is a market disruption event. In such a case, one needs to follow the rules on Market Disruption Events detailed later in this document. The second is when calculating the RealVol Real-Time Index. Demand Derivatives will use the most recent underlying price throughout the trading day (“Today”) as the URP, even though the market may not have closed. Such an exception occurs on today’s value only and not for any other day in the past.

## Defined Days

As noted, there are six time frames for RealVol Indices: 1, 5, 21, 63, 126, and 252 trading days. However, because of the potential for a market disruption event (MDE) where the market never opens and hence never closes, the actual number of trading days may be less than expected. When this occurs, the number of actual trading days ("trading days") will be less than the expected days ("Defined Days"). Note: We cannot count a non-trading day's volatility as zero just because a market disruption event eliminates an entire day of trading.

For further clarification: weekends are not trading days; holidays are not trading days; a regularly scheduled trading day is a "trading day." However, a trading day where the market and all of its surrogate markets do not open, and hence cannot close, causes the number of trading days to be less than the number of Defined Days for purposes of the RealVol Index calculation.

### Design of RealVol Daily Formula

#### Mean Set to Zero

The RealVol Daily Formula starts with the traditional formula for standard deviation and modifies it in a few key ways. First, we set the mean to zero in order to provide “movement regardless of direction” instead of “movement about a mean or trend.” Doing so makes hedging easier for options traders and corresponds to the formula used for variance swaps and volatility swaps in the over-the-counter market.

#### Annualization Factor

Second, Demand Derivatives sets the annualization factor to a constant. The constant value of 252 represents the number of trading days in a typical year in the U.S. Because of the vagaries of the calendar in any particular year and/or the holiday schedules in any particular country, the actual number of trading days may be slightly higher or lower than 252. However, it is preferable to have one approximate constant than to have a variety of exact, but different, values.

#### Degrees of Freedom

Third, “degrees of freedom” is a term in statistics used to extrapolate from a sample of data to the entire dataset. Since the intent is to provide the exact realized volatility over a specific period and not to extrapolate that sample dataset to the entire history of trading, Demand Derivatives sets the degrees of freedom to zero.

#### Dollar and Cents Construct

Finally, the result is typically a value less than 1.00. Demand Derivatives multiplies the result by 100 in order to bring the values to a more intuitive “dollars and cents” construct. For example, the annualized realized volatility of an equity index may be 0.20. Often, traders would quote this number as 20%. Demand Derivatives would disseminate the index value as 20.00.

### RealVol Daily Formula

#### Formula 1

$$VOL = 100 \sqrt{\frac{252}{n} \sum_{t=1}^n R_t^2}$$

Where:

$VOL$  = daily (i.e., close-to-close) realized volatility  
 252 = a constant representing the approximate number of trading days in a year  
 $t$  = a counter representing each trading day  
 $n$  = number of trading days in the measurement time frame  
 $R_t$  = continuously compounded daily returns as calculated by:

$$R_t = Ln \frac{P_t}{P_{t-1}}$$

$Ln$  = natural logarithm  
 $P_t$  = URP ("closing price") at day  $t$   
 $P_{t-1}$  = URP at the trading day immediately preceding day  $t$

### RealVol Real-Time Formula

#### Formula 2

$$VOL = 100 \sqrt{\frac{252}{n} \left[ \frac{86,400 - s}{86,400} R_1^2 + \sum_{t=2}^n R_t^2 + R_{n+1}^2 \right]}$$

Where:

$VOL$  = real-time realized volatility  
 86,400 = number of seconds in a day  
 $n+1$  = today  
 $s$  = number of seconds up to the current moment in time of the current day ( $n+1$ ) beginning from the time of the most recent market close (day  $n$ ), excluding intervening weekend days and holidays, but not excluding market disruption events (MDEs)  
 $R_1$  = return for the first day (day 1) of the period (from close day zero to close day 1)  
 $R_{n+1}$  = partial return (the return using the current underlying price and the URP of the prior day).

Note: For clarification, the non-italic "R" denotes partial return; all other returns, in italics, are full-day returns.

### RealVol Overnight/Intraday Formula

#### Formula 3

$$DVOL = \sqrt{\frac{252}{n} \sum_{i=1}^n \left( \ln \left( \frac{UOP_i}{URP_{i-1}} \right) \right)^2 + \frac{252\pi}{8} \sum_{i=1}^n \left( \frac{\ln \left( \frac{UHP_i}{ULP_i} \right)}{n} \right)^2}$$

Where:

$DVOL$  = overnight/intraday realized volatility  
 $UOP_i$  = today's underlying open price  
 $URP_{i-1}$  = yesterday's underlying reference price (i.e., yesterday's close)  
 $UHP_i$  = today's highest underlying price  
 $ULP_i$  = today's lowest underlying price

The full paper may be accessed via the following link:  
<http://www.realvol.com/DVOLPaper.pdf>

## RealVol Correlation Formula

### Formula 4

$$VCOR = \frac{252 \sum_{i=1}^n R_i R_{RVOL_i}}{n VOL VOV}$$

Where:

$VCOR$  = realized correlation between an underlying and its volatility

$R_{RVOL_i}$  = the return of the VOL index each day

## RealVol Variance Formula

### Formula 5

$$VAR = 100 \frac{252}{n} \sum_{t=1}^n R_t^2$$

## RealVol Rough Vol Formula

### Formula 6

The Rough Vol model is described in detail in the following papers.

<http://www.realvol.com/RVOL1Paper.pdf>

<http://www.realvol.com/RVOL2Paper.pdf>

Our preliminary research has shown that the Rough Vol model approximates future realized volatility more accurately than the market (based on implied volatility).

## RealVol HARK Vol Formula

### Formula 7

The HARK model is a proprietary model developed exclusively for Demand Derivatives. Its roots are derived from the HAR model as described in the following paper.

<http://www.realvol.com/HVOLPaper.pdf>

Our preliminary research has shown that the HARK Vol model approximates future realized volatility more accurately than the market (based on implied volatility).

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## Formula Special Considerations

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### Time of Day

For the purposes of the above, the calculation of the number of seconds, ("s"), within the current trading day does not necessarily start at the beginning of the day (midnight), but rather at the closing time of the market on the previous trading day. For example, the stock market in the U.S. closes at 4:00 PM Eastern Time (16:00 on a 24-hour clock). Therefore, the end of the trading day is 4:00 PM and "tomorrow's" trading day begins immediately afterwards.

If the current time is 7:00 PM (19:00), the current day's second count is 10,800 (60 seconds per minute x 60 minutes per hour x 3 hours after the market's close). To continue with this example, three hours is 3/24 (or 0.125 expressed as a decimal) of a whole day. Similarly, 10,800 seconds is 10,800/86,400 (or the same 0.125 expressed as a decimal) of a whole day.

Note: It makes no difference if the market is indeed open 24 hours. If the market opens at, say, 9:30 AM (09:30) the following morning, this is 17.5 hours after the market closed on the previous day; thus, 17.5/24 hours, or 63,000/86,400 seconds, or 0.7292 weight for the current day's (n+1) most recent return would be used in the calculation.

### Non-Trading Days

Since there are weekends, holidays, and potential market disruption events that could occur, it is important to know how the calculation of the RealVol Indices will be affected by these non-trading, or partially trading, days.

1. In the case of a weekend day, there is no URP or possible calculation of a return, so weekend days will be ignored. In essence, the close-to-close formula will continue as if the non-trading weekend days never existed. No RealVol Index will be calculated or disseminated on weekend days.
2. In the case of holidays, there is no URP or possible calculation of a return, so those days will be ignored. In essence, the close-to-close formula will continue as if the non-trading holiday never existed. No RealVol Index will be calculated or disseminated on holidays.
3. In the case of a partial day Market Disruption Event (MDE), please see the section entitled Process for Market Disruption Events.

4. If the MDE prevents the trading or calculation of any URP for the entire day, no return calculation is possible. However, the VOL and VOV Indices will continue to be calculated and published (if publication is possible). In order to keep the same rolling set of daily returns moving through time, the VOL Indices cannot simply ignore the originally scheduled trading day that did not occur. Thus, it will use the same set of data normally scheduled for the rolling 5-day, 21-day, 63-day, 126-day, and 252-day versions, but will compensate for the missing day's returns by lowering the value of n by the full number of days of the MDE. See Exhibit 3 for a table outlining the day counts prior to, throughout, and after a full day's MDE. Note: for the 1-day VOL, it is not possible to calculate a volatility. Therefore, on those days, VOL will not exist.

For example, if today is a scheduled trading day, but the market could not open, and hence the market could not close, the normal 21-day VOL Index will be published as a 20-day VOL Index for the period during which the MDE coincides with the normal 21-day returns schedule. In essence, the first day will be dropped as we perform the normal roll process, but the 21<sup>st</sup> day will not be added because there is no URP available "today."

It should be noted that the MDE will continue to affect the calculation of all historical Indices even after the MDE has passed. This is because every Realized Volatility calculation has a look-back period, and if that period coincides with the MDE, it will continue to affect the calculation in the same manner. For example, suppose that an MDE event occurred yesterday and the market opens normally today. All RealVol Indices greater than the 1-day time frame will be calculated and disseminated for today's normal trading day. However, the number of returns will be reduced by 1. Please see Exhibit 3 showing the value of n through time.



- The orange highlighted day indicates a day where all indices will be published but where the underlying URP, and hence that daily return, will not.

Now that the formula is defined, the URPs may need adjusting to remove any “contrived,” “artificial,” or “non-normal” volatility that may exist (collectively called “phantom volatility”). There are many examples of how such phantom volatility is possible.

### Stocks Paying a Dividend

Many companies pay a dividend to holders of company stock. If such a stock were the underlying, and the company paid a dividend, typically its stock would drop by the value of the dividend on the ex-dividend day. This is rational because no value has been created by such an event. A dividend merely transfers value from the company to the shareholder. For example, a company worth \$99 per share that pays a \$1 dividend should be valued afterwards at \$98 (assuming all other factors have remained unchanged). The shareholder still has \$99 of value (the stock worth \$98 plus \$1 in cash). However, if one is measuring the day-to-day return of this stock for volatility calculation purposes, the stock price just dropped roughly 1%. If the formula for calculating realized volatility did not take such an artificial drop into account, the resulting volatility would be misleading.

### Stock Splits, Reverse Splits, Stock Dividends

Another example of phantom volatility would be if the underlying were a stock and the company goes through a stock split, reverse split, or stock dividend (for simplicity, all will be referred to as a “stock split” as the concept is the same regardless). Assume that, in this case, the number of shares doubles. Such an action obviously increases the number of shares outstanding but should not affect the value of the company. This is because in order to keep the value of the company a constant, the share price must drop in half to compensate. However, since realized volatility is based only on price changes, and since the price changed substantially (artificially dropped in half), the realized volatility of the stock would be increased to an artificially large value. If the formula for calculating realized volatility did not take such an artificial drop into account, the resulting volatility would be misleading.

### Expiring Underlyings

If the underlying is an instrument that expires, such as a futures contract, there is an adjustment required in order not to introduce phantom volatility that is truly not present.

There comes a point in the life of a futures contract where the front-month contract expires. And, it is logical to assume that the previously deferred month, which now becomes the new front month, then becomes the contract upon which further returns are based. Futures

contracts are, as their name implies, based on a prediction — a forecast of a future event. As such, their values are predicated upon many factors, and so it is natural to assume that when one futures contract expires, the next one, chronologically, which may not expire until one month, two months, or perhaps three months, later, may differ in price from the recently expired one. Clearly, when one endeavors to calculate the RealVol Indices based on an underlying, and that underlying is a futures contract, such a “jump” in successive URPs could be problematic. Consider the following.

Suppose that an underlying March futures contract has just expired at a price of 100. Suppose, further, that, at that very moment of expiration, the deferred underlying June contract is trading at 102. Finally, suppose that, in the next day’s trading, the underlying June futures contract remains unchanged and closes once again at 102. In the calculation of a continuing series of closing-settlement returns, one might use the underlying March contract until it stops trading. In this case, the final price is 100. For the next trading day, there is zero inter-day volatility, because the market is unchanged, and yet the new closing reference point would be 102 — that of the underlying June contract. In the calculation process, were one to simply “roll” the return calculation from underlying March into underlying June, there would be the appearance of a two-point jump in the reference prices, from 100 to 102, implying some daily realized volatility when, in fact, there is none.

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## Adjusting for Phantom Volatility

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### Adjustment for Dividends

If the underlying is a security and the security pays a dividend, the security's price needs to be adjusted to account for any dividend paid. Therefore, to properly calculate realized volatility, add the dividend to the URP (be it closing or intraday) on the ex-dividend day before calculating a return for that day. The end result removes phantom volatility caused by a contrived situation of a paid dividend. The following day, the adjustment is not needed or carried through, and the calculation reverts to simply the return for the day, using the unadjusted, actual URPs.

### Dividends

Normally, "today's" price is the same as the URP.

$$P_t = URP$$

In the event of a dividend, the price used as the URP will be adjusted by the dividend payment as outlined in Formula 7.

#### Formula 7

$$P_t = URP_{t-1} + Dividend_t$$

Where:

$URP_{t-1}$  = Underlying Reference Price from "yesterday"

$Dividend_t$  = The per-share dividend to be paid to shareholders of record on "today's" ex-dividend day.

### Adjustment for Stock Splits

Stock splits, therefore, can introduce phantom volatility. To eliminate such phantom volatility, the daily or intra-day return would be adjusted by multiplying the share price on its stock-split day by the ratio of the new number of shares divided by the original number of shares. For example, if the number of original shares was 1,000 and a stock split such that every share becomes two shares, the ratio is  $2,000/1,000 = 2$ . Hence, the price on stock-split day would need to be multiplied by two as well in order to eliminate phantom volatility caused because of a stock split. The following day, the adjustment is not needed or carried through, and the calculation reverts to simply the return for the day, using the unadjusted, actual URPs.

#### Formula 8

$$P_t = URP_{t-1} \frac{\text{Number of shares after split}_t}{\text{Number of shares before split}_{t-1}}$$

## Adjustment for Expiring Underlyings

To address this potential problem, the rollover method proceeds on the day following the expiration of the underlying futures contract by calculating the daily return using the URP from the next-to-expire futures contract only. For example, suppose it is expiration day of the underlying March contract. The return calculation for expiration day will use the URP for the day before and the URP for the current day to calculate a daily return (this is the standard process with no adjustment or special process needed). However, to ensure continuity of pricing, without the possibility of a "false jump" on the following day, one would immediately resort to referencing the URPs of the underlying June contract only. In other words, when calculating the next day's daily return, the URP for the prior day (the day that the March contract expired) will be the June URP, not the March URP. Doing so would avoid any possibility of a gap or jump in price due solely to the underlying roll process. In still other words, while the March contract is "alive," its daily returns are used solely; on the day after the underlying March expiration, the daily returns of the underlying June contract are used solely. This process is repeated over and over as we move through time and the sequential underlying futures contracts expire.

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## Process for Market Disruption Events

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### Surrogate Markets

When a market disruption event occurs in the primary market such that it affects the close, a surrogate market may be used. The reasoning is that it makes sense to use another similar market to get a close estimate of the return on the MDE day rather than to resort to the alternative of simply eliminating that day from the calculation. Of course, if all surrogate markets are similarly affected, there is no choice but to eliminate the day's return calculation (as outlined in the section entitled "Non-Trading Days, point 4").

### Using Returns versus URPs

Sometimes a surrogate market is not in the same units or has a typical premium or discount to the primary market. Despite any of these pricing issues, the surrogate market's *returns* will always be used (not just replacing the day's surrogate market URP). To clarify: the surrogate market's return is calculated by using today's closing price and yesterday's closing price of the surrogate market exclusively. No URP of the primary market is comingled with URP's of the surrogate market during an MDE.

### Complete Closure of the Primary Market

If the primary market cannot open, and hence cannot close, the following steps shall be taken:

1. Look to the surrogate market(s) (in the order listed).
2. If a surrogate market is also affected, but is only partially so, such that any surrogate market closes normally, that surrogate market shall be used to provide the daily return.
3. If none of the surrogate markets close normally but at least one is open for a portion of the day, the surrogate market whose last trade or last update is the latest shall be used as the closing day's URP and used for the MDE day's return. Note: For purposes of determining the latest time, the next-in-line surrogate market must be open for at least five minutes beyond the surrogate market's closure/disruption in order for it to be considered "open for a longer portion of the day."
4. If none of the surrogate markets open, there is no choice but to eliminate this MDE day from the daily realized volatility calculation (as outlined in the section entitled "Non-Trading Days, point 4"). Note: Eliminating the underlying return because of an MDE does not eliminate the RealVol Index calculation for that day.

### Partial Closure of the Primary Market (but Closing Normally)

If the primary market has an MDE but closes normally, no special action is necessary (the closing price of the primary market shall be used as the URP).

### Partial Closure of the Primary Market (and not Closing Normally)

If the primary market has an MDE and cannot close normally, the surrogate markets shall be referenced in an attempt to provide the MDE day's returns according to the below steps:

1. Look to the surrogate market(s) (in the order listed).
2. If a surrogate market is also affected, but is only partially so, such that any surrogate market closes normally, that surrogate market shall be used to provide the daily return.
3. If none of the surrogate markets close normally but at least one is open for a longer portion of the day than the primary market, the surrogate market whose last trade or last update is the latest shall be used as the closing day's URP and used for the MDE day's return. Note: For purposes of determining the latest time, the surrogate market must be open for at least five minutes beyond the primary market's closure/disruption in order for it to be considered "open for a longer portion of the day."
4. If none of the surrogate markets trading or pricing hours extend beyond the primary market's hours, the primary market's latest or last price shall be used as the URP for the day.

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## Calculation Examples of VOL Daily Indices

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The following is a detailed example of how the index calculation is computed. Instead of providing details on every index, we will provide details only on the VOL indices (three versions of 21-, 63-, and 252-day calculations).

### VOLm Index

The following is a step-by-step description of the daily VOLm Index-calculation methodology. Note: The following refer to Exhibit 4. Also note that the spreadsheet contains data through 1 March 2019.

- In column A are trading dates. Weekends and holidays are removed because the calculation ignores those days.
- In column B are the closing prices of the underlying corresponding to the date in column A. In this particular case, we use the SPDR® S&P 500 ETF<sup>1</sup> (symbol SPY) as the underlying, but any underlying with a daily closing price is possible.
- In column C is “today’s” closing price ( $T_0$ ) divided by “yesterday’s” closing price ( $T_{-1}$ ). Specifically, cell C4 has the formula “=B4/B3”. This cell is copied down the column such that cell C5 has the formula “=B5/B4”, etc. (Excel changes the reference cells automatically as the initial formula is copied down the spreadsheet.)
- In column D is the continuously compounded return of each value in column C. The formula in cell D4 is “=LN(C4)”. Again, this formula is copied down the column and Excel changes the references automatically to the next cell such that cell D5 will now contain the formula “=LN(C5)”, etc.
- In column E is the squared return. The formula in column E4 is “=D4^2”. Note: the “^” symbol means “to raise the variable to the power of.” So,  $D4^2$  is equal to the square of D4 (or  $D4 * D4$ ). Again, this cell is copied down the column and Excel changes the relative references.
- In column F is the sum of the previous 21 days’ returns. The formula in cell F24 is “=SUM(E4:E24)”. In Excel, the “SUM” function adds all items within the parentheses. And, in this case, “E4:E24” is the accepted notation to include everything in the cell starting with E4 and continuing in order through E24. Therefore, the “=SUM(E4:E24)” is equivalent to “=SUM(E4,E5,E6,E7,...,E24)”, and in mathematical notation, this is equivalent to  $E4 + E5 + E6 + E7 + \dots + E24$ . The formula gets copied down the page in a similar manner as described above. Excel automatically

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<sup>1</sup> S&P® and SPDR® are registered trademarks of Standard & Poor's Financial Services LLC.

changes the formula reference each time such that cell F25 would have the formula “=SUM(E5:E25)”, etc.

- In column G is the average of the 21 days’ returns. To get the average, divide each value in column F by 21. The formula in G24 is “=F24/21”. Again, this value is copied down the column.
- In column H is the annualization factor. To annualize the value, just multiply each value in column G by 252 (a constant used to approximate the number of trading days in a year). The formula in cell H24 is “=G24\*252”. Note: The symbol “\*” means to multiply. Again, copy the formula down the column.
- In column I is the square root of each value in column H. The formula in cell I24 is “=SQRT(H24)”. “SQRT” is the built-in function in Excel that takes the square root of the number within the parentheses. Again, this cell is copied down the column.
- In column J is the removal of the percentage sign by multiplying the result in column I by 100. The formula in cell J24 is “=I24\*100”.
- In column K is the “Daily VOLmSPY Index.” The formula in K24 is “=J24”.

As you will notice, it takes 22 days of closing prices to calculate 21 days of returns. And, the first 21 returns are needed in order to calculate the very first VOLm index value. Therefore, whatever URPs are used, a VOLm index cannot produce its very first value until 21 days have passed. However, please note that this is a one-time issue. The SPY Index has been available since 1993. Therefore, the VOLmSPY Index can be calculated back to 2 March 1993 (approximately one month after the launch of the underlying) and would be continuous since then.

For clarification, the example in Exhibit 4 shows that the VOLm Index started in 3 January 2019. This was done simply for explanatory purposes. The VOLmSPY Index values easily could have been calculated for January 2019 with data from December 2018. And, data for Dec 2018 could have been calculated with data in Nov 2018, and so on, back in time all the way to 3 March 1993.

## VOLq Index

The methodology for the VOLq Index is exactly the same as the VOLm calculation methodology except that there are 63 days of returns. The only changes to the spreadsheet would be in columns F and G. In column F the Sum function would need to encompass 63 days (not 21), and in column G the denominator would need to change from 21 to 63. Similarly to the 21-day example, in this case, the calculations in column F and G cannot start until row 66, as one cannot calculate the very first VOLq value until after the first 63 days of underlying data are available. All other calculations remain the same.

## VOLy Index

To calculate the VOLy Index, the methodology remains the same except column F and G need to change. In this case, column F needs to encompass 252 days of data, and the denominator in column G needs to change to 252. Similarly to the 21-day and 63-day examples, one cannot calculate the very first VOLy value until after the first 252 days of underlying data are available. All other calculations remain the same.

Exhibit 4

	A	B	C	D	E	F	G	H	I	J	K
1	SPY ETF	Close	Close "Today" divided by close "Yesterday"	Continuously Compounded Returns	Returns Squared	Sum 21 Squared Returns	Average All Returns by Dividing by 21	Annualize by Multiplying by 252	Take Square Root of Result	Remove Percentage Sign by Multiplying by 100	VOLmSPY Daily Value
2	Date	Close	Return	Compounding	Return^2	21 Day Sum	Average	Annualize	Sqrt	Index	VOL
3	2-Jan-19	250.18									
4	3-Jan-19	244.21	97.61%	-2.42%	0.000583						
5	4-Jan-19	252.39	103.35%	3.29%	0.001086						
6	7-Jan-19	254.38	100.79%	0.79%	0.000062						
7	8-Jan-19	256.77	100.94%	0.94%	0.000087						
8	9-Jan-19	257.97	100.47%	0.47%	0.000022						
9	10-Jan-19	258.88	100.35%	0.35%	0.000012						
10	11-Jan-19	258.98	100.04%	0.04%	0.000000						
11	14-Jan-19	257.40	99.39%	-0.61%	0.000037						
12	15-Jan-19	260.35	101.15%	1.14%	0.000130						
13	16-Jan-19	260.98	100.24%	0.24%	0.000006						
14	17-Jan-19	262.96	100.76%	0.76%	0.000057						
15	18-Jan-19	266.46	101.33%	1.32%	0.000175						
16	22-Jan-19	262.86	98.65%	-1.36%	0.000185						
17	23-Jan-19	263.41	100.21%	0.21%	0.000004						
18	24-Jan-19	263.55	100.05%	0.05%	0.000000						
19	25-Jan-19	265.78	100.85%	0.84%	0.000071						
20	28-Jan-19	263.76	99.24%	-0.76%	0.000058						
21	29-Jan-19	263.41	99.87%	-0.13%	0.000002						
22	30-Jan-19	267.58	101.58%	1.57%	0.000247						
23	31-Jan-19	269.93	100.88%	0.87%	0.000076						
24	1-Feb-19	270.06	100.05%	0.05%	0.000000	0.002901	0.000138	0.034816658	0.186592	18.66	18.66
25	4-Feb-19	271.96	100.70%	0.70%	0.000049	0.002367	0.000113	0.028406594	0.168543	16.85	16.85
26	5-Feb-19	273.10	100.42%	0.42%	0.000017	0.001299	0.000062	0.015590541	0.124862	12.49	12.49
27	6-Feb-19	272.74	99.87%	-0.13%	0.000002	0.001239	0.000059	0.01487125	0.121948	12.19	12.19
28	7-Feb-19	270.14	99.05%	-0.96%	0.000092	0.001244	0.000059	0.014922828	0.122159	12.22	12.22
29	8-Feb-19	270.47	100.12%	0.12%	0.000001	0.001223	0.000058	0.014679834	0.121160	12.12	12.12
30	11-Feb-19	270.62	100.06%	0.06%	0.000000	0.001211	0.000058	0.014534723	0.120560	12.06	12.06
31	12-Feb-19	274.10	101.29%	1.28%	0.000163	0.001374	0.000065	0.016492083	0.128422	12.84	12.84
32	13-Feb-19	274.99	100.32%	0.32%	0.000011	0.001347	0.000064	0.016168789	0.127157	12.72	12.72
33	14-Feb-19	274.38	99.78%	-0.22%	0.000005	0.001222	0.000058	0.014669641	0.121118	12.11	12.11
34	15-Feb-19	277.37	101.09%	1.08%	0.000117	0.001334	0.000064	0.016009171	0.126527	12.65	12.65
35	19-Feb-19	277.85	100.17%	0.17%	0.000003	0.001280	0.000061	0.015359553	0.123934	12.39	12.39
36	20-Feb-19	278.41	100.20%	0.20%	0.000004	0.001109	0.000053	0.013310279	0.115370	11.54	11.54
37	21-Feb-19	277.42	99.64%	-0.36%	0.000013	0.000937	0.000045	0.011242185	0.106029	10.60	10.60
38	22-Feb-19	279.14	100.62%	0.62%	0.000038	0.000971	0.000046	0.01164819	0.107927	10.79	10.79
39	25-Feb-19	279.52	100.14%	0.14%	0.000002	0.000972	0.000046	0.011667008	0.108014	10.80	10.80
40	26-Feb-19	279.32	99.93%	-0.07%	0.000001	0.000902	0.000043	0.010821218	0.104025	10.40	10.40
41	27-Feb-19	279.20	99.96%	-0.04%	0.000000	0.000844	0.000040	0.010124967	0.100623	10.06	10.06
42	28-Feb-19	278.68	99.81%	-0.19%	0.000003	0.000845	0.000040	0.010145514	0.100725	10.07	10.07
43	1-Mar-19	280.42	100.62%	0.62%	0.000039	0.000637	0.000030	0.007649998	0.087464	8.75	8.75

## Shorthand

As just explained, Exhibit 4 takes the reader through each step one at a time. While such a process may be needed the first time to teach the concepts, combining the steps make the spreadsheet much more compact. Exhibit 5 shows how to combine the formulas for ease of use.

Cell C4 contains the formula  $=\text{LN}(B4/B3)^2$ . Cell D24 has the formula  $=100*\text{SQRT}(\text{SUM}(C4:C24)*252/21)$ . Again, copy those cells down the spreadsheet and the cell references will automatically update to the correct relative reference. One should notice that the end result is the same.

If the time frame is not 21 trading days, the formula needs to be edited to accommodate the different calculation period. The final example will use a one week or 5-day calculation period (not shown). Cell C4 contains the formula  $=\text{LN}(B4/B3)^2$ . Cell D8 has the formula  $=100*\text{SQRT}(\text{SUM}(C4:C8)*252/5)$ . Again, copy those cells down the page to process all underlying data accordingly.

Exhibit 5

	A	B	C	D
1	SPY ETF		Interim Calculation	VOLmSPY Daily Value
2	Date	Close	Squared Returns	VOL
3	2-Jan-19	250.18		
4	3-Jan-19	244.21	0.000583	
5	4-Jan-19	252.39	0.001086	
6	7-Jan-19	254.38	0.000062	
7	8-Jan-19	256.77	0.000087	
8	9-Jan-19	257.97	0.000022	
9	10-Jan-19	258.88	0.000012	
10	11-Jan-19	258.98	0.000000	
11	14-Jan-19	257.40	0.000037	
12	15-Jan-19	260.35	0.000130	
13	16-Jan-19	260.98	0.000006	
14	17-Jan-19	262.96	0.000057	
15	18-Jan-19	266.46	0.000175	
16	22-Jan-19	262.86	0.000185	
17	23-Jan-19	263.41	0.000004	
18	24-Jan-19	263.55	0.000000	
19	25-Jan-19	265.78	0.000071	
20	28-Jan-19	263.76	0.000058	
21	29-Jan-19	263.41	0.000002	
22	30-Jan-19	267.58	0.000247	
23	31-Jan-19	269.93	0.000076	
24	1-Feb-19	270.06	0.000000	18.66
25	4-Feb-19	271.96	0.000049	16.85
26	5-Feb-19	273.10	0.000017	12.49
27	6-Feb-19	272.74	0.000002	12.19
28	7-Feb-19	270.14	0.000092	12.22
29	8-Feb-19	270.47	0.000001	12.12
30	11-Feb-19	270.62	0.000000	12.06
31	12-Feb-19	274.10	0.000163	12.84
32	13-Feb-19	274.99	0.000011	12.72
33	14-Feb-19	274.38	0.000005	12.11
34	15-Feb-19	277.37	0.000117	12.65
35	19-Feb-19	277.85	0.000003	12.39
36	20-Feb-19	278.41	0.000004	11.54
37	21-Feb-19	277.42	0.000013	10.60
38	22-Feb-19	279.14	0.000038	10.79
39	25-Feb-19	279.52	0.000002	10.80
40	26-Feb-19	279.32	0.000001	10.40
41	27-Feb-19	279.20	0.000000	10.06
42	28-Feb-19	278.68	0.000003	10.07
43	1-Mar-19	280.42	0.000039	8.75

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## Notice

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