

**Specifications
of
RealVol[™] Indices**

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Overview

Volatility Defined

In the financial markets, all investments have risk. Because of risk, there also exists opportunity. Many people automatically think of risk as “bad,” or synonymous with *risk of loss*. However, the true definition of risk encompasses not only the potential for loss, but the potential for gain as well. One of the ways to measure price risk is known as “volatility.” This is why volatility is so important in the endeavor of investing and risk management. In its simplest description, volatility is “magnitude of movement regardless of direction.” In other words, volatility deals only with price change, but is indifferent as to whether that change is up or down.

Measurement, Not Observation

One of the key issues regarding realized volatility is that it is *measured* not *observed*. In other words, one cannot look at a particular price and determine what volatility that represents. The only way to calculate realized volatility is to get prices over time and then use a realized volatility formula to calculate the movement of prices for that time frame. This is a key concept because all realized volatility formulas must use historical prices in order to calculate a result.

For example, if the evening news reports that the price of gold is currently \$1,000 per ounce, it has expressed only the *price* of gold. The current price says nothing about how much gold had to move to get to \$1,000. Did it just move down from \$2,000? Or, did it just move up from \$999? One cannot tell unless previous (historical) data are also used. Thus, saying that the price of gold is now \$1,000 tells us nothing about its volatility, because we lack a historical perspective.

RealVol Indices

This document describes the process of creating RealVol Indices to measure realized volatility and related statistical methods in a standardized manner on a daily and real-time basis. The one real-time version is trademarked under the name *RealVol Real-Time Index*. The other versions will be updated once per day (“daily”) and trademarked under the name *RealVol Daily Indices*. The flagship set of RealVol Indices attempt to measure the movement, regardless of direction, of some underlying, over a predefined time frame. All RealVol Indices are expected to be published on a multitude of underlyings.

Daily Versus Real-Time

No index, that VolX is aware of, currently has both daily and real-time versions simultaneously. Most indices are updated on a real-time, or nearly real-time, basis. Some are calculated daily. But, none have both. Why the distinction in this case? Typically, volatility is measured on a daily basis only. Therefore, VolX created the RealVol Daily Indices to correspond to the standard of

using only daily (i.e., closing) underlying reference prices (“URPs”). The VOL Index price on any particular day is the value that will be used to settle tradable RealVol Instruments.

However, traders often demand indices that are updated more frequently. The problem is how to furnish a real-time version for a daily volatility index. VolX solved this conundrum by taking today’s URP before the close (numbered day #22) and weighting it by the proportion of time through today’s trading day, then using the remaining weight for the first URP (day #1). In essence, the first day of the period (22 trading days prior, day #1) and the last day of the period (today, day #22) will have a combined weight of 100% in total, while the days in between will have a weight of 100% each. In this way, the 22 returns will be weighted as if there are only 21 returns and the RealVol Index will, therefore, be updating throughout the day as today’s URP changes. Note: the RealVol *Real-Time* Formula results in exactly the same value as the RealVol *Daily* Formula at the instant when the market closes.

Design of RealVol Real-Time Formula

All of the design elements described for the RealVol Daily Formula are the same for the RealVol Real-Time Formula. To convert from a daily to a real-time value, one needs to start with the RealVol Daily Formula, then incorporate the current underlying price and a weighting scheme. Doing so provides continuous updates throughout the trading day and delivers to traders a useful, real-time indication of the up-to-the-moment 21-day daily realized volatility. At VolX, it was decided to convert only the 1-month VOL Index (VOLm) to a real-time index (VOL). Essentially, VOL measures a constant 21-day realized volatility even while we are within the new, most recent, day (“Today”).

For instance, if we are 80% through the current day (n+1), we will use the most up-to-the-moment URP to calculate the partial (80%) day’s return (n+1) from yesterday’s URP (n). Then we consider the very first day in the calculation period and weight that whole day’s return by 20% (100%–80%). In this manner, we still have the weight of 21-day realized volatility at any moment in time even though there are actually 22 returns – 20% weight on day 1, 80% weight on day 22, and full weights for days 2 through 21 (for a total weight of 21 days of returns).

Note: While the partial return of the current day is self-weighting, and therefore requires no additional coefficient, the self-weighted portion of the current day is nonetheless required to be calculated so as to apply the proper remaining weight to the full day 1 return. In order to calculate the weight of the current day, the current time each day is measured to the closest second. Since there are 86,400 seconds in a day, the current time and the number of seconds in a day are used in the RealVol Real-Time Formula to calculate the daily weight to be applied to day 1.

When the time of day equals the close of today (n+1), the weight of the return of day n+1 is now 100%, while the weight of the return of day 1 is 0%. Thus, with its weight of zero, the return of

the original day 1 drops out of the calculation. The original day 2 now becomes the new day 1 and all other days get renumbered as well. The RealVol Real-Time Formula at this very instant in time (the close at 4:00 PM ET in our example) simplifies to the RealVol Daily Formula. The instant after the market closes, we begin anew, with the returns renumbered, such that there are again only 21 returns, with the new trading day having a weighted return as day 22.

Note that the new “day” begins immediately after the close of the market on a particular day (ignoring holidays and weekends), not necessarily at the end of the day (i.e., midnight).

RealVol Daily Indices (Overview)

There are 40 RealVol Daily Indices: seven types in each of six time frames. Note: we may use the abbreviation “vol” to mean realized volatility.

Seven Types

1. Realized vol (VOL™)
2. Realized vol of vol (VOV™)
3. Overnight/Intraday Realized vol (DVOL™)
4. Correlation (underlying vs. vol) (VCOR™)
5. Realized Variance (VAR™)
6. RFSV (“Rough”) Forecast vol (RVOL™)
7. HARK Forecast vol (HVOL™)

Six Time Frames

1. 1 day
2. 1 week (5 trading days)
3. 1 month (21 trading days)
4. 1 quarter (63 trading days)
5. 1 half-year (126 trading days)
6. 1 year (252 trading days)

Note: 1-day and 1-week time frames are not statistically meaningful. Still, it may be interesting to reference such results. Also, the 1-day and 1-week time frames are meaningless for Correlation; therefore, those two time frames are omitted.

RealVol Daily Indices (Table)

Base Symbol	Description	Day	Week (5-day)	Month (21-day)	Quarter (63-day)	Half Year (126-day)	Year (252-day)
VOL	Realized Volatility	VOLd	VOLw	VOLm	VOLq	VOLh	VOLy
VOV	Realized Volatility of Realized Volatility	VOVd	VOVw	VOVm	VOVq	VOVh	VOVy
DVOL	Overnight/ Intraday “Daily” Realized Volatility	DVOLd	DVOLw	DVOLm	DVOLq	DVOLh	DVOLy
VCOR	Correlation of Underlying vs. VOL	N/A	N/A	VCORm	VCORq	VCORh	VCORy
VAR	Realized Variance	VARd	VARw	VARm	VARq	VARh	VARy
RVOL	RFSV “Rough” Model Forecast of VOL	RVOLd	RVOLw	RVOLm	RVOLq	RVOLh	RVOLy
HVOL	HARK Model Forecast of VOL	HVOLd	HVOLw	HVOLm	HVOLq	HVOLh	HVOLy

RealVol Daily Indices (Details)

VOL Indices

Realized volatility indices (VOL) measure the interday (close-to-close) realized volatility of an underlying using the RealVol Daily Formula. One can think of VOL as a measure of interday price risk of an underlying. It is especially useful as a guide for traders of options in the underlying and for those trading volatility instruments.

VOV Indices

Realized volatility of realized volatility indices (VOV) measure the realized volatility of VOL using the RealVol Daily Formula a second time on the same data. Trading instruments on realized volatility compels one to discover the risk of those instruments. To do so, one needs to calculate the vol of vol. The VOV indices attempt to quantify such risk and can be especially useful as a guide to options traders of realized volatility instruments.

Note: The second iteration through the data is always performed on a 1-month (21-day) basis, regardless of the time frame of VOL.

DVOL Indices

Overnight/intraday realized volatility indices (DVOL) measure the overnight/intraday realized volatility from previous day's close and today's open, high, and low data of an underlying using the RealVol Overnight/Intraday Formula. One can think of DVOL as a measure of overnight/intraday price risk of an underlying. It can be especially useful as a guide to timing trades in the underlying. This model was developed by Robert Krause, CEO of VolX, and Professor Steven Satchell, University of Sydney.

VCOR Indices

Correlation indices (VCOR) measure the correlation between the underlying and its VOL. Use VCOR for insight into the relationship between the underlying and its realized volatility.

VAR Indices

Realized variance indices (VAR) measure the interday (close-to-close) realized variance of an underlying using the RealVar Daily Formula. One can think of VAR as another measure of interday price risk of an underlying. Variances are easier to sum, average, and combine because the result is linear as opposed to volatility, which is a curve function.

RVOL Indices

RFSV forecasts of realized volatility indices (RVOL) project future interday (close-to-close) realized volatility of an underlying using a RFSV ("Rough" vol) model. These models are provided courtesy of James Gatheral of Baruch College.

According to the Rough Volatility model, the log volatility process is well approximated by fractional Brownian motion (fBm) with a Hurst parameter H close to zero. One can think of this process as representing the combined influence of all traders in the market with trading frequencies ranging from very low to very high. Although it has only two parameters, H and the volatility of volatility, the Rough Volatility model successfully replicates both

observed features of the time series of volatility such as volatility clustering and slow decay of the volatility autocorrelation function and also observed features of the implied volatility surface such as power-law decay of at-the-money volatility skew.

There is a well-known formula due to Nuzman and Poor that constructs forecasts of fBm from the time-series of fBm. The RVOL forecast exploits this formula to express forecast future volatility in terms of the time series of daily realized volatilities; the logarithm of each historical realized volatility observation is weighted by a factor that decays hyperbolically both in the time elapsed since the historical observation and in the time horizon of the forecast. This amounts to a more sophisticated version of the trader's rule that volatility over a given future time interval (for example 3 months) is best estimated by looking at historical volatility over the equivalent time interval (again 3 months in this example). RVOL forecasts are both easy to implement and accurate.

HVOL Indices

HARK forecasts of realized volatility indices (HVOL) project future interday (close-to-close) realized volatility of an underlying using a HARK model. These models are provided courtesy of Fulvio Corsi, professor at Università Ca' Foscari Venezia.

The HARK model is a Heterogeneous Autoregressive (HAR) model cast into a Kalman filter framework. The HAR model views the market as composed of traders with heterogeneous time horizons that perceive and cause volatility at different frequencies. This heterogeneous market structure leads to a volatility dynamic where future volatility depends on past volatilities realized over different time horizons and can thus be modeled by the so-called Heterogeneous Autoregressive (HAR) model. In spite of its simplicity, the HAR model is able to fully capture the high persistence of volatility dynamics observed in empirical data. The combination of ease of implementation and a very accurate fit of financial volatility time series have made the HAR models very popular in the financial econometrics community.

HARK is a dynamic extension of the HAR model with leverage effects obtained by casting the HAR model into a Kalman filter framework. In this way the parameters of the HAR are continuously and optimally updated by the Kalman filter according to the statistical properties of the volatility input. This means that the lookback period of the model adapts to the changing conditions of the markets. Therefore, the HARK model is able to combine the persistence and accurate fitting of the asymmetric HAR model with the flexibility and fast adaptation of the Kalman filter.

In view of their simplicity and impressive empirical success, the Rough and HARK volatility models have sparked great interest from both academics and practitioners.

Terms

Realized Volatility

Realized volatility is the “actual volatility,” “statistical volatility,” or “asset volatility” that the underlying has displayed over a specific period. The term “realized volatility” is very closely related to “standard deviation.” Realized volatility is a specific form of standard deviation. If one were to use daily *returns* of an underlying (instead of actual *prices*) and annualize the results, standard deviation becomes realized volatility. VolX uses a modified version of the standard deviation formula. We will refer to the RealVol version of realized volatility as “realized volatility,” “realized vol,” or simply “vol.”

Implied Volatility

There is another type of volatility known as “*implied volatility*.” Implied volatility is based on the relative expensiveness of associated options premiums. Implied volatility is a completely different approach to expressing volatility and often differs in value from realized volatility. The intricacies of implied volatility are beyond the scope of this document; hence, when we use the term “volatility,” we are referring to realized volatility.

Underlying

The term “underlying” is used as a noun to mean just about anything that has a daily price. For example, the underlying can be a physical asset, security, futures contract, index, currency, bond, swap, measurement, etc.

Underlying Reference Price (URP)

We will use the term Underlying Reference Price (URP) to refer to the actual daily closing price that the underlying has displayed, or will go on to display. The URP is the “closing,” “final,” or “settlement” price for the day. The URP is an especially attractive value for calculating Realized Volatility because of its ease of use, transparency, and wide dissemination to market participants. Roughly speaking, therefore, the underlying is the “asset,” while the URP is the “closing price.”

However, there are two exceptions: The first is when there is a market disruption event. In such a case, one needs to follow the rules on Market Disruption Events detailed later in this document. The second is when calculating the RealVol Real-Time Index. VolX will use the most recent underlying price throughout the trading day (“Today”) as the URP, even though the market may not have closed. Such an exception occurs on today’s value only and not for any other day in the past.

Defined Days

As noted, there are six time frames for RealVol Indices: 1 trading day, 5 trading days, 21 trading days, 63 trading days, 126 trading days, and 252 trading days. However, because of the potential for a market disruption event (MDE) where the market never opens and hence never closes, the actual number of trading days may be less than expected. When this occurs, the number of actual trading days (“trading days”) will be less than the expected days (“Defined Days”). Note: We cannot count a non-trading day’s volatility as zero just because a market disruption event eliminates an entire day of trading.

For further clarification: weekends are not trading days; holidays are not trading days; a regularly scheduled trading day is a “trading day.” However, a trading day where the market and all of its surrogate markets do not open, and hence cannot close, causes the number of trading days to be less than the number of Defined Days for purposes of the RealVol Index calculation.

Formulas

Design of RealVol Daily Formula

Mean Set to Zero

The RealVol Daily Formula starts with the traditional formula for standard deviation and modifies it in a few key ways. First, we set the mean to zero in order to provide “movement regardless of direction” instead of “movement about a mean or trend.” Doing so makes hedging easier for options traders and corresponds to the formula used for variance swaps and volatility swaps in the over-the-counter market.

Annualization Factor

Second, VolX sets the annualization factor to a constant. The constant value of 252 represents the number of trading days in a typical year in the U.S. Because of the vagaries of the calendar in any particular year and/or the holiday schedules in any particular country, the actual number of trading days may be slightly higher or lower than 252. However, it is preferable to have one approximate constant than to have a variety of exact, but different, values.

Degrees of Freedom

Third, “degrees of freedom” is a term in statistics used to extrapolate from a sample of data to the entire dataset. Since the intent is to provide the exact realized volatility over a specific period and not to extrapolate that sample dataset to the entire history of trading, VolX sets the degrees of freedom to zero.

Dollar and Cents Construct

Finally, the result is typically a value less than 1.00. VolX multiplies the result by 100 in order to bring the values to a more intuitive “dollars and cents” construct. For example, the annualized realized volatility of an equity index may be 0.20. Often, traders would quote this number as 20%. VolX would disseminate the index value as 20.00.

RealVol Daily Formula

Formula 1

$$VOL = 100 \sqrt{\frac{252}{n} \sum_{t=1}^n R_t^2}$$

Where:

VOL = daily (i.e., close-to-close) realized volatility

252 = a constant representing the approximate number of trading days in a year

t = a counter representing each trading day

n = number of trading days in the measurement time frame

R_t = continuously compounded daily returns as calculated by:

$$R_t = Ln \frac{P_t}{P_{t-1}}$$

Ln = natural logarithm

P_t = URP (the underlying reference price or “closing price”) on day t

P_{t-1} = URP on the trading day immediately preceding day t

RealVol Real-Time Formula (available only in the 1-month time frame)

Formula 2

$$VOL = 100 \sqrt{\frac{252}{n} \left[\frac{86,400 - s}{86,400} R_1^2 + \sum_{t=2}^n R_t^2 + R_{n+1}^2 \right]}$$

Where:

VOL = real-time realized volatility

86,400 = number of seconds in a day

$n+1$ = today (normally day 22)

s = number of seconds up to the current moment in time of the current day ($n+1$, normally day 22) beginning from the time of the most recent market close (normally day 21), excluding intervening weekend days and holidays, but not excluding market disruption events (MDEs)

R_1 = return for the first day (day 1) of the period (from close day 0 to close day 1)

R_{n+1} = partial return (the return using the current underlying price and the URP of the prior day).

Note: For clarification, the non-italic “R” denotes partial return; all other returns, in italics, are full-day returns.

RealVol Overnight/Intraday Formula

Formula 3

$$DVOL = \sqrt{\frac{252}{n} \sum_{i=1}^n \left(\ln \left(\frac{UOP_i}{URP_{i-1}} \right) \right)^2 + \frac{252\pi}{8} \sum_{i=1}^n \left(\frac{\ln \left(\frac{UHP_i}{ULP_i} \right)}{n} \right)^2}$$

Where:

$DVOL$ = overnight/intraday realized volatility

UOP_i = today’s underlying open price

URP_{i-1} = yesterday’s underlying reference price (i.e., yesterday’s close)

UHP_i = today’s highest underlying price

ULP_i = today’s lowest underlying price

RealVol Correlation Formula

Formula 4

$$VCOR = \frac{\sum_{i=1}^n R_i R_{VOL_i}}{\sqrt{\sum_{i=1}^n R_i^2} \sqrt{\sum_{i=1}^n R_{VOL_i}^2}}$$

Where:

$VCOR$ = realized correlation between an underlying and its volatility

R_{VOL_i} = the return of the VOL index each day

RealVol Variance Formula

Formula 6

$$VAR = 100 \frac{252}{n} \sum_{t=1}^n R_t^2$$

Formula Special Considerations

RealVol Real-Time Index (VOL)

The flagship index, the RealVol *Real-Time* Index (VOL), is available in one timeframe only: 1-month (21-day), realized volatility.

It works as follows, if the time today is 75% of the way through the current trading day (normally day 22), then weight the partial day's return by 75% (this partial return is self-weighting) and weight the full day's return from normally 22 days prior (day 1) by 25% (this term needs to be specifically weighted). All other days are full-weight, full-day (close-to-close) returns. In this manner and in most every case, the index encompasses 22 data points but the weight of exactly 21 trading days at any moment throughout the trading day.

Note: It makes no difference if the market is indeed open 24 hours. If the market opens at, say, 9:30 AM (09:30) the following morning, this is 17.5 hours after the market closed on the previous day. Thus the weighting scheme begins the trading day with 17.5/24 hours, or 63,000/86,400 seconds. In addition, normally the calculation encompasses 22 days of data while within today's trading day and 21 days of data at exactly the close. This day count can be adversely affected by any full-day market disruption event (as described in the following section).

Non-Trading Days

Since there are weekends, holidays, and potential market disruption events that could occur, it is important to know how the calculation of the RealVol Indices will be affected by these non-trading, or partially trading, days.

1. In the case of a weekend day, there is no URP or possible calculation of a return, so weekend days will be ignored. In essence, the close-to-close formula will continue as if the non-trading weekend days never existed. No RealVol Index will be calculated or disseminated on weekend days.
2. In the case of holidays, there is no URP or possible calculation of a return, so those days will be ignored. In essence, the close-to-close formula will continue as if the non-trading holiday never existed. No RealVol Index will be calculated or disseminated on holidays.
3. In the case of a partial day Market Disruption Event (MDE), please see the section entitled Process for Market Disruption Events.
4. If the MDE prevents the trading or calculation of any URP for the entire day, no return calculation is possible. However, the VOL and VOV Indices will continue to be calculated and published (if publication is possible). In order to keep the same rolling set of daily returns moving through time, the VOL Indices cannot simply ignore the originally scheduled trading day that did not occur. Thus, it will use the same set of data normally scheduled for the rolling 5-day, 21-day, 63-day, 126-day, and 252-day versions, but will compensate for the missing day's returns by lowering the value of n by the full number of days of the MDE. See Exhibit 1 for a table outlining the day counts prior to, throughout, and after, a full day's MDE.

For example, if today is a scheduled trading day, but the market could not open, and hence the market could not close, the normal 21-day VOL Index will be published as 20-day VOL Index for the period during which the MDE coincides with the normal 21-day returns schedule. In essence, the first day will be dropped as we perform the normal roll process, but the 21st day will not be added because there is no URP available "today." In the potential of an MDE that spans multiple days, such that neither the primary market nor any of the surrogate markets open, and hence cannot close, the algorithm will continue to reduce the number of days in the calculation period to as few as one day for all indices ("Min Calculation Period").

If, indeed, an MDE causes the underlying to lack pricing for less than the Min Calculation Period the RealVol Indices will remain the same until such time as the URPs are again available. Subsequent days will be added, but no days removed, until the Defined Days are restored to their full number of expected trading days. It should be noted that the MDE will continue to affect the calculation of VOL and VOV Indices even after the MDE has passed (again, unless the Minimum Calculation Period has been exceeded). This is because every Realized Volatility calculation has a look-back period, and if that period coincides with the MDE, it will continue to affect the calculation in the same manner.

For example, suppose that an MDE event occurred yesterday and the market opens normally today. All RealVol Indices will be calculated and disseminated for today's normal trading day. However, in the example of 1-month Indices, the number of returns will be reduced by 1 until 20 days beyond the MDE day. Please see Exhibit 1 showing the value of n through time.

Phantom Volatility

Now that the formula is defined, the URPs may need adjusting to remove any “contrived,” “artificial,” or “non-normal” volatility that may exist (collectively called “phantom volatility”). There are many examples of how such phantom volatility is possible.

Stocks Paying a Dividend

Many companies pay a dividend to holders of company stock. If such a stock were the underlying, and the company paid a dividend, typically its stock would drop by the value of the dividend on the ex-dividend day. This is rational because no value has been created by such an event. A dividend merely transfers value from the company to the shareholder. For example, a company worth \$99 per share that pays a \$1 dividend should be valued afterwards at \$98 (assuming all other factors have remained unchanged). The shareholder still has \$99 of value (the stock worth \$98 plus \$1 in cash). However, if one is measuring the day-to-day return of this stock for volatility calculation purposes, the stock price just dropped roughly 1%. If the formula for calculating realized volatility did not take such an artificial drop into account, and then correct for it, the resulting volatility would be misleading.

Stock Splits, Reverse Splits, Stock Dividends

Another example of phantom volatility would be if the underlying were a stock and the company were to go through a stock split, reverse split, or stock dividend (for simplicity, all will be referred to as a “stock split” as the concept is the same regardless). Assume that, in this case, the number of shares doubles. Such an action obviously increases the number of shares outstanding but should not affect the value of the company. This is because in order to keep the value of the company a constant, the share price must be halved to compensate. However, since realized volatility is based only on price changes, and since the price changed substantially (artificially dropped 50%), the realized volatility of the stock would be increased to an artificially large value. If the formula for calculating realized volatility did not take such an artificial drop into account, and then correct for it, the resulting volatility would be misleading.

Expiring Underlyings

If the underlying is an instrument that expires, such as a futures contract, there is an adjustment required in order not to introduce phantom volatility that is truly not present.

There comes a point in the life of a futures contract where the front-month contract expires. And, it is logical to assume that the previously deferred month, which now becomes the new front month, then becomes the contract upon which further returns are based. Futures contracts are, as their name implies, based on a prediction — a forecast of a future event. As such, their values are predicated upon many factors, and so it is natural to assume that when one futures contract expires, the next one, chronologically,

which may not expire until one month, two months, or perhaps three months, later, may differ in price from the recently expired one. Clearly, when one endeavors to calculate the RealVol Indices based on an underlying, and that underlying is a futures contract, such a “jump” in successive URPs could be problematic. Consider the following.

Suppose that an underlying March futures contract has just expired at a price of 100. Suppose, further, that, at that very moment of expiration, the deferred underlying June contract is trading at 102. Finally, suppose that, in the next day’s trading, the underlying June futures contract remains unchanged and closes once again at 102. In the calculation of a continuing series of closing-settlement returns, one might use the underlying March contract until it stops trading. In this case, the final price is 100. For the next trading day, there is zero interday volatility, because the market is unchanged, and yet the new closing reference point would be 102 — that of the underlying June contract. In the calculation process, were one to simply “roll” the return calculation from underlying March into underlying June, there would be the appearance of a two-point jump in the reference prices, from 100 to 102, implying some daily realized volatility when, in fact, there is none.

A Deliverable Futures Contract as an Underlying

When the underlying is a deliverable commodity futures contract (instead of being cash settled), the roll from one contract to the next is performed early (rolling just prior to the delivery month). This is done because the unfettered price discovery process can be adversely affected during the delivery process. Hence, for many commodities and energy products, RealVol indices use pre-delivery rolls.

Short-Term Interest Rate Futures

When short-term rates approach zero, calculating returns can be problematic as denominators cannot be zero. Therefore, the accepted method in the marketplace to address this concern is to use a normal distribution rather than a lognormal distribution when the interest rate is at or below 1.00%. This corresponds to findings of several academic papers, which studied the issue. It also corresponds to the accepted method found in the marketplace.

An Index as the Underlying

If the underlying is an index, no adjustments shall be made except if the index goes through a normalization event (i.e., the index is rebased). Any treatment of dividends, or lack thereof, or the treatment of stock splits, stock dividends, etc., is in the domain of the company creating and calculating the index. RealVol will not “look through” the index to the underlying stocks and eliminate any phantom volatility. So, for example, if one wants information on the realized volatility of the S&P 500 with or without dividends, one could look to the RealVol Index on the S&P 500 Index or the S&P 500 Total Return Index as appropriate.

Adjusting for Phantom Volatility

Adjustment for Dividends

If the underlying is a security and the security pays a dividend, the security's price needs to be adjusted to account for any dividend paid. Therefore, to properly calculate realized volatility, subtract the dividend from the URP on the day prior to ex-dividend day before calculating a return for ex-dividend day. The end result removes phantom volatility caused by a contrived situation of a paid dividend. The day after ex-dividend day, the adjustment is not needed or carried through, and the calculation reverts to simply the return for the following day, using the unadjusted, actual URPs.

To clarify: An index is not a security. Therefore, no adjustment for dividends shall pass through from the stocks that compose the index to the calculation of realized volatility. For example, if one wanted the realized volatility of the S&P 500 index without adding back the dividends, one would simply use the index values as published each day by Standard and Poor's. If one wanted the realized volatility of the S&P 500 index with dividends reinvested, one would simply use the S&P 500 Total Return Index as the underlying.

Dividends

Normally, the closing price on any given day is the same as the URP.

$$P_t = URP_t$$

In the event of a dividend paid on a security, the price used as the URP on the day prior to ex-dividend day (URP_{XDD-1}) will be adjusted by the dividend payment as outlined in Formula 7.

Formula 7

$$P_{t-1} = URP_{XDD-1} - Dividend_{XDD}$$

Where:

XDD = Ex-dividend day

URP_{XDD-1} = Underlying Reference Price on the day prior to ex-dividend day

$Dividend_{XDD}$ = The per-share declared dividend to be paid to shareholders of record on ex-dividend day.

Adjustment for Stock Splits, Reverse Splits, and Stock Dividends

Stock splits, reverse splits, and stock dividends, also can introduce phantom volatility. To eliminate such phantom volatility, the daily or intraday return would be adjusted by multiplying the share price on the day before its ex-event day by the ratio of the original number of shares divided by the new number of shares. For example, if the number of original shares was 1,000 and a stock split such that every share becomes two shares, the ratio is $\frac{1}{2}$ ($1,000/2,000$). Hence, the price on the day before the ex-event day would need to be multiplied by one-half in order to eliminate

phantom volatility caused by a stock split. For subsequent days, the adjustment is not needed or carried through, and the calculation reverts to simply the return for the day, using the unadjusted, actual URPs.

Formula 8

$$P_{t-1} = URP_{XED-1} \frac{\text{Number of shares before split}_{XED-1}}{\text{Number of shares after split}_{XED}}$$

Where:

XED = Ex-event day (the day that the split, reverse split, or stock dividend affects the shareholders of record).

URP_{XED-1} = Underlying reference price on the day prior to the ex-event day

Index Adjustments

Similar to a security that goes through a contrived event such as a stock split or reverse stock split, sometimes an index will go through an event often called a “normalization process.” In such a case, the index is “normalized” or “rebased” to another index level. Since this is a contrived event that causes phantom volatility, the effects of such an action need to be removed from the “normal” variability of the index.

The adjustment is the same process as Formula 8 except that the index adjustments replace the security adjustments. Note: There will be no “look through” to the components of an index in order to adjust for phantom volatility. Adjustments will be made only at the index level if needed as outlined in the previous paragraph.

Adjustment for Expiring Underlyings

To address the potential problem of expiring underlyings such as futures, a “rollover” method is required. The rollover method will change depending on whether the RealVol Index is a “shorter-term” index or a “longer-term” index.

In the case of the shorter-term RealVol Indices (1-day, 1-week, and 1-month), each day’s index value includes only URPs/returns from the same-expiration futures contract. In contrast, longer-term RealVol Indices may include URPs/returns from more than one futures contract expiration.

To help bring more clarity to the above, first let us discuss when the roll is performed. In all cases (long-term or short-term), calculations occur normally until the nearest-to-expire futures contract expires (Day T) using URPs/returns from that nearby referenced futures contract (FUTs A). On the day after expiration of the nearest futures, when the second-to-expire contract becomes the nearest-to-expire contract, a roll must be performed in order for the RealVol Index to continue uninterrupted (Day T+1) using URPs/returns from the now nearest-to-expire futures contract (FUTs B). This process is handled differently for short-term RealVol Indices as compared to long-term RealVol Indices.

For short-term RealVol Indices, Day T is calculated normally using returns from FUTs A. On Day T+1, the data from the entire realized volatility period is based on the returns of FUTs B. In other words, the RealVol Index on Day T+1 includes no FUTs A returns, only FUTs B returns for the entire Calculation Period (21-days back in the case of monthly realized volatility).

In contrast, longer-term RealVol Indices have a slightly different roll process. On Day T, again, the RealVol Index is calculated normally using URPs/returns from FUTs A. On Day T+1, the RealVol Index includes the URPs/return from FUTs B on Day T+1, however all previous URPs/returns come from the URPs/returns of FUTs A. In other words, Day T+1 uses FUTs B return; Day T uses FUTs A return, and all prior days before Day T use FUTs A returns.

Such a roll change is necessary for two reasons: For shorter-term indices, if instruments were ever traded on the 1-month RealVol Index, it would be advantageous to have each expiration include only returns from the same underlying futures expiration. For longer-term indices, expiration of a particular futures contract may not be listed as long as 252 days in advance. Even if it were listed, the contract may not be liquid enough to provide reliable pricing. For these reasons, when the underlying is a futures contract, the roll due to futures expirations is treated differently depending on whether the RealVol Index is longer-term or shorter-term.

Adjustment for Deliverable Futures as an Underlying

If the underlying is a deliverable commodity futures contract, rolling will occur immediately after the last day of the month prior to the delivery month. URPs within the delivery month are ignored.

Adjustment When the Underlying Is Quoted as an Interest Rate

Because interest rates can fall to zero and perhaps even lower into negative territory, an adjustment needs to take place. If an attempt is made to calculate a return where the denominator is zero or one needs to take the natural log of zero, either result is undefined. To alleviate this issue, VolX will substitute a formula calculating change (normal distribution) instead of calculating returns (lognormal distribution) when interest rates are below 1%.

Process for Market Disruption Events

Surrogate Markets

When a market disruption event occurs in the primary market such that it affects the close, a surrogate market's returns may be used. The reasoning is that it makes sense to use another similar market to get a close estimate of the return on the MDE day rather than to resort to the alternative of simply eliminating that day from the calculation. Of course, if all surrogate markets are

similarly affected, there is no choice but to eliminate the day's return calculation (as outlined in the section entitled "Non-Trading Days, sub-point 4").

Using Returns versus URPs

Sometimes a surrogate market is not in the same units or has a typical premium or discount to the primary market. Despite any of these pricing issues, the surrogate market's *returns* will always be used (not just replacing the day's surrogate market URP). To clarify: the surrogate market's return is calculated by using today's closing price and yesterday's closing price of the surrogate market exclusively. No URP of the primary market is comingled with URPs of the surrogate market during an MDE. When the primary market opens again, the returns will again revert to the primary market.

Complete Closure of the Primary Market

If the primary market cannot open, and hence cannot close, the following steps shall be taken:

1. Look to the surrogate market(s) (in the order listed) for a surrogate return.
2. If a surrogate market is also affected, but is only partially so, such that any surrogate market closes normally, that surrogate market shall be used to provide the daily return.
3. If none of the surrogate markets close normally but at least one is open for a portion of the day, the surrogate market whose last trade or last update is the latest shall be used as the closing day's URP and used for the MDE day's return.
4. If none of the surrogate markets open, there is no choice but to eliminate this MDE day from the daily realized volatility calculation (as outlined in the section entitled "Non-Trading Days, sub-point 4"). Note: Eliminating the underlying return because of an MDE does not eliminate the RealVol Index calculation for that day.

Partial Closure of the Primary Market (but Closing Normally)

If the primary market has an MDE but closes normally, no special action is necessary (the closing price of the primary market shall be used as the URP).

Partial Closure of the Primary Market (and not Closing Normally)

If the primary market has an MDE and cannot close normally, the surrogate markets shall be referenced in an attempt to provide the MDE day's returns according to the below steps:

1. Look to the surrogate market(s) (in the order listed) for a surrogate return.
2. If a surrogate market is also affected, but is only partially so, such that any surrogate market closes normally, that surrogate market shall be used to provide the daily return.
3. If none of the surrogate markets close normally but at least one is open for a longer portion of the day than the

primary market, the surrogate market whose last trade or last update is the latest shall be used as the closing day's URP and used for the MDE day's return.

4. If none of the surrogate markets trading or pricing hours extend beyond the primary market's hours, the primary market's latest or last price shall be used as the URP for the day.

Calculation Examples of VOL Daily Indices

The following is a detailed example of how the index calculation is computed. Instead of providing details on every index, we will provide details on only the 1-month VOL Index (VOLm).

VOLm Index

The following is a step-by-step description of the daily 1-month VOL Index-calculation methodology. Note: The following refer to Exhibit 2. Also note that the spreadsheet contains data through 1 March 2012.

- In column A are trading dates. Weekends and holidays are removed because the calculation ignores those days.
- In column B are the closing prices of the underlying corresponding to the date in column A. In this particular case, we use the S&P 500 Index[®] as the underlying, but any underlying with a daily closing price is possible.
- In column C is "today's" closing price divided by "yesterday's" closing price. Specifically, cell C2 has the formula " $=B2/B1$ ". This cell is copied down the column such that cell C3 has the formula " $=B3/B2$ ", etc. (Excel changes the reference cells automatically as the initial formula is copied down the spreadsheet.)
- In column D is the continuously compounded return of each value in column C. The formula in cell D2 is " $=LN(C2)$ ". Again, this formula is copied down the column and Excel changes the references automatically to the next cell such that cell D3 will now contain the formula " $=LN(C3)$ ", etc.
- In column E is the squared return. The formula in column E2 is " $=D2^2$ ". Note: the " $^$ " symbol means "to raise the variable to the power of." So, $D2^2$ is equal to the square of D2 (or $D2 * D2$). Again, this cell is copied down the column and Excel changes the relative references.
- In column F is the sum of the previous 21 days' returns. The formula in cell F22 is " $=SUM(E2:E22)$ ". In Excel, the "SUM" function adds all items within the parentheses. And, in this case, "E2:E22" is the accepted notation to include everything in the cell starting with E2 and continuing in order through E22. Therefore, the " $=SUM(E2:E22)$ " is equivalent to " $=SUM(E2,E3,E4,E5,...,E22)$ ", and in mathematical notation, this is equivalent to $E2+E3+E4+E5+...+E22$. The formula gets copied down the page in a similar manner as described above. Excel automatically changes the formula reference each time such that cell F23 would have the formula " $=SUM(E3:E23)$ ", etc.

- In column G is the average of the 21 days' returns. To get the average, divide each value in column F by 21. The formula in G22 is " $=F22/21$ ". Again, this value is copied down the column.
- In column H is the annualization factor. To annualize the value, just multiply each value in column G by 252 (the approximate number of trading days in a year). The formula in cell H22 is " $=G22*252$ ". Note: The symbol "*" means to multiply. Again, copy the formula down the column.
- In column I is the square root of each value in column H. The formula in cell I22 is " $=SQRT(H22)$ ". "SQRT" is the built-in function in Excel that takes the square root of the number within the parentheses. Again, this cell is copied down the column.
- In column J is the removal of the percentage sign by multiplying the result in column I by 100. The formula in cell J22 is " $=I22*100$ ".
- In column K is the "Daily 1-month VOL U.S. 500 Index." The formula in K22 is " $=J22$ ".

As you will notice, it takes 22 days of closing prices to calculate 21 days of returns. And, the first 21 returns are needed in order to calculate the very first 1-month VOL Index value. Therefore, whatever URPs are used, a 1-month VOL Index cannot produce its very first value until 21 days have passed. However, please note that this is a one-time issue. The S&P 500 Index has been available in its current form since 1957 (the author believes that the index was actually started in January 1957). Therefore, the 1-month VOL Index based on the underlying of S&P 500 Index can be calculated back to February 1957 (approximately one month after the launch of the underlying) and would be continuous since then.

For clarification, the example in Exhibit 2 shows that the 1-month VOL Index started in Feb 2012. This was done simply for explanatory purposes. The 1-month VOL Index values easily could have been calculated for January 2012 with data from December 2011. And, data for Dec 2011 could have been calculated with data in Nov 2011, and so on, back in time all the way to February 1957.

1-Quarter VOL Index (VOLq)

The methodology for the 1-quarter VOL Index (VOLq) is exactly the same as the 1-month VOL Index calculation methodology except that there are normally 63 days of returns instead of the normal 21 days of returns. The only changes to the spreadsheet would be in columns F and G. In column F the Sum function would need to encompass 63 days, and in column G the denominator would need to change from 21 to 63. Similarly to the 21-day example, in this case, one cannot calculate the very first 1-quarter VOL Index value until after the first 63 days of underlying data are available. All other calculations remain the same.

1-Half Year VOL Index (VOLh)

The methodology for the 1-half year VOL Index (VOLh) is exactly the same as the 1-month VOL Index calculation methodology except that there are normally 126 days of returns instead of the normal 21 days of returns. The only changes to the spreadsheet would be in columns F and G. In column F the Sum function would need to encompass 126 days, and in column G the denominator would need to change from 21 to 126. Similarly to the 21-day example, in this case, one cannot calculate the very first 1-half year VOL Index value until after the first 126 days of underlying data are available. All other calculations remain the same.

1-Year VOL Index (VOLy)

Similar to the 1-Quarter VOL Index just described, the methodology remains the same except column F and G need to change. In this case, column F needs to encompass 252 days of data, and the denominator in column G needs to change to 252. Similarly to the 21-day and 63-day examples, in this case, one cannot calculate the very first 1-year VOL Index value until after the first 252 days of underlying data are available. All other calculations remain the same.

Exhibit 2

A	B	C	D	E	F	G	H	I	J	K
Date	S&P 500 Index® Close	Close "Today" divided by Close "Yesterday" Step 1	Continuously Compound Return Step 2	Square Return Step 3	Sum 21 Returns Step 4	Average All Returns by Dividing by 21 Step 5	Annualize by Multiplying by 252 Step 6	Take Square Root of Result Step 7	Remove Percentage Sign by Multiplying by 100 Step 8	VolX 1RVOL US 500 Index Step 9
1	30-Dec-11	1257.60								
2	3-Jan-12	1277.06	1.0155	1.5355%	0.0236%					
3	4-Jan-12	1277.30	1.0002	0.0188%	0.0000%					
4	5-Jan-12	1281.06	1.0029	0.2939%	0.0009%					
5	6-Jan-12	1277.81	0.9975	-0.2540%	0.0006%					
6	9-Jan-12	1280.70	1.0023	0.2259%	0.0005%					
7	10-Jan-12	1292.08	1.0089	0.8847%	0.0078%					
8	11-Jan-12	1292.48	1.0003	0.0310%	0.0000%					
9	12-Jan-12	1295.50	1.0023	0.2334%	0.0005%					
10	13-Jan-12	1289.09	0.9951	-0.4960%	0.0025%					
11	17-Jan-12	1293.67	1.0036	0.3547%	0.0013%					
12	18-Jan-12	1308.04	1.0111	1.1047%	0.0122%					
13	19-Jan-12	1314.50	1.0049	0.4927%	0.0024%					
14	20-Jan-12	1315.38	1.0007	0.0669%	0.0000%					
15	23-Jan-12	1316.00	1.0005	0.0471%	0.0000%					
16	24-Jan-12	1314.65	0.9990	-0.1026%	0.0001%					
17	25-Jan-12	1326.06	1.0087	0.8642%	0.0075%					
18	26-Jan-12	1318.43	0.9942	-0.5771%	0.0033%					
19	27-Jan-12	1316.33	0.9984	-0.1594%	0.0003%					
20	30-Jan-12	1313.01	0.9975	-0.2525%	0.0006%					
21	31-Jan-12	1312.41	0.9995	-0.0457%	0.0000%					
22	1-Feb-12	1324.09	1.0089	0.8860%	0.0079%	0.0721%	0.0034%	0.8648%	9.2993%	9.30
23	2-Feb-12	1325.54	1.0011	0.1094%	0.0001%	0.0486%	0.0023%	0.5833%	7.6371%	7.64
24	3-Feb-12	1344.90	1.0146	1.4500%	0.0210%	0.0696%	0.0033%	0.8355%	9.1406%	9.14
25	6-Feb-12	1344.33	0.9996	-0.0424%	0.0000%	0.0688%	0.0033%	0.8254%	9.0849%	9.08
26	7-Feb-12	1347.05	1.0020	0.2021%	0.0004%	0.0685%	0.0033%	0.8225%	9.0692%	9.07
27	8-Feb-12	1349.96	1.0022	0.2158%	0.0005%	0.0685%	0.0033%	0.8220%	9.0663%	9.07
28	9-Feb-12	1351.95	1.0015	0.1473%	0.0002%	0.0609%	0.0029%	0.7307%	8.5479%	8.55
29	10-Feb-12	1342.64	0.9931	-0.6910%	0.0048%	0.0657%	0.0031%	0.7879%	8.8761%	8.88
30	13-Feb-12	1351.77	1.0068	0.6777%	0.0046%	0.0697%	0.0033%	0.8364%	9.1457%	9.15
31	14-Feb-12	1350.50	0.9991	-0.0940%	0.0001%	0.0673%	0.0032%	0.8080%	8.9887%	8.99
32	15-Feb-12	1343.23	0.9946	-0.5398%	0.0029%	0.0690%	0.0033%	0.8278%	9.0985%	9.10
33	16-Feb-12	1358.04	1.0110	1.0965%	0.0120%	0.0688%	0.0033%	0.8257%	9.0867%	9.09
34	17-Feb-12	1361.23	1.0023	0.2346%	0.0006%	0.0669%	0.0032%	0.8032%	8.9620%	8.96
35	21-Feb-12	1362.21	1.0007	0.0720%	0.0001%	0.0669%	0.0032%	0.8033%	8.9624%	8.96
36	22-Feb-12	1357.66	0.9967	-0.3346%	0.0011%	0.0680%	0.0032%	0.8164%	9.0356%	9.04
37	23-Feb-12	1363.46	1.0043	0.4263%	0.0018%	0.0697%	0.0033%	0.8370%	9.1486%	9.15
38	24-Feb-12	1365.74	1.0017	0.1671%	0.0003%	0.0626%	0.0030%	0.7507%	8.6643%	8.66
39	27-Feb-12	1367.59	1.0014	0.1354%	0.0002%	0.0594%	0.0028%	0.7129%	8.4436%	8.44
40	28-Feb-12	1372.18	1.0034	0.3351%	0.0011%	0.0603%	0.0029%	0.7234%	8.5051%	8.51
41	29-Feb-12	1365.68	0.9953	-0.4748%	0.0023%	0.0619%	0.0029%	0.7428%	8.6184%	8.62
42	1-Mar-12	1374.09	1.0062	0.6139%	0.0038%	0.0656%	0.0031%	0.7877%	8.8755%	8.88

RealVol SPY Indices

Underlying

SPDR® S&P 500® ETF Trust (symbol SPY).

Closing Time

RealVol® SPY calculations are based on close at 4:00 PM (16:00) ET each business day.

Underlying Reference Price

The URP shall be the close of SPY on the primary market at NYSE Arca.

Surrogates

In the event that a market disruption event affects the functioning of the primary market, the following instruments' prices shall be referenced and their return may be used as a surrogate return (in the order listed).

1. The 4:00 PM ET close of another market trading SPY in the U.S. (using the close at the exchange with the highest volume)
2. The S&P 500® index as published by Standard and Poor's.
3. The nearest to 4:00 PM ET trade of the E-mini S&P 500 futures contract as traded at the CME.

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