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RealVol Futures Overlay on an S&P 500® Portfolio

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1. Introduction

The Volatility Exchange™ (VolX®) plans to launch futures and options contracts based upon the realized volatility of U.S. equity indices. The futures version is named RealVol™ futures (VOL), which settle to the RealVol indices known generically as RVOL™. The concept is both similar and dissimilar to the popular VIX® index and products marketed by the CBOE®. The two versions are similar in the notion that both VolX and CBOE are trying to provide volatility products to the marketplace. They are dissimilar because the VIX index and consequently VIX futures are based on implied volatility (the relative cost of options) while the RVOL index and consequently VOLs are based on realized volatility (the actual, historical movement of the underlying index).

VOLs are exchange-tradable instruments that function similarly to a forward-starting over-the-counter volatility swap. They are expected to be launched on U.S. equity indices in 2013 and will come in two varieties: a 1-month calculation period of realized volatility (1VOL™) and a 3-month calculation period of realized volatility (3VOL™)[i]. The goal of this paper is to demonstrate how a VOL overlay can enhance the return and/or reduce the standard deviation of an equity portfolio. We chose the S&P 500 Total Return Index on the assumption that VolX will roll out products based upon this index.

Because VOLs are not yet available for trading it is impossible to perform an empirical study with actual traded prices and daily settlement values. However, historical options data is available. Assuming that VOLs would be priced in line with such option prices, we can use a pricing model to determine theoretical values of VOLs based on traded options prices. Our study covers a time period beginning in 1990 corresponding to the start date of our options dataset. We chose the Heston Model to provide theoretical volatility-swap-like pricing. However, because a volatility swap normally starts its measurement period immediately after creation, we needed an additional adjustment to compensate for the forward-starting nature of VOLs. For that adjustment, we applied a root-mean-square (RMS) formula to the result of the Heston Model.

This paper is organized as follows: First, the methodology for determining theoretical VOL prices is introduced. Second, the performance of simple buy-and-hold S&P 500 portfolios (S&P) are compared to portfolios called the buy-and-hold overlay (B&H) consisting of S&P plus a small allocation to 1VOL in a continuous process. Third, we consider a more active approach by adjusting our exposure to 1VOL based on a simple moving-average criterion. Finally, we consider a slightly more complex strategy that combines a long-term moving average with a shorter-term moving average in a variety of active allocation methodologies for both the 1VOL and 3VOL.

2. The Valuation of RealVol Futures

We calculate theoretical VOL prices using the Heston model (which is ultimately based on associated option premiums), along with an adjustment based on the to-date realized volatility (named partial realized volatility, or PVOL™), as appropriate. Of course, one must realize that there are many market forces that affect valuations; consequently, VOLs may not have traded at their model-derived theoretical volatility values. For example, by using the Heston model, we are assuming that this is the correct model to price VOLs. All models make assumptions about the state of the market, and the Heston model is no exception. Those assumptions may not be valid at all times. We are also assuming that the inputs (associated option premiums) are correct or are trading at their theoretically correct value. As we know, such assumptions are not always valid. Finally, even if the model had come up with the perfect theoretical value, such a price may not have actually provided the trader with a profit. Therefore, one should not base a trade or strategy solely on a model-derived assumption of theoretical pricing.

As stated, VOLs are essentially forward starting, exchange-traded volatility swaps that ultimately expire to the daily (i.e., close-to-close) realized volatility of the underlying, as calculated by the VolX daily formula. Thus, the prices of the RealVol futures and volatility swaps should be similar. However, unlike volatility swaps, which normally start their realized volatility calculation period (CP or calculation period) immediately upon creation, VOLs start their CP on a pre-designated date, typically in the future. Therefore, while all VOLs have a CP, most are listed for trading prior to the start of their CP. In other words, most VOLs have both an anticipatory period (AP, the period between initial listing and the start of the CP) and a calculation period. Before we can describe how to determine a theoretical price for VOLs, we will show how to price a volatility swap using the Heston model. Then, a root-mean-square (RMS) calculation will adjust the theoretical value for the fixed start-date feature of VOLs.

The VolX daily formula is as follows:

$$Vol = 100 \times \sqrt{\frac{252}{n}\sum_{t-1}^{n} R_t^2}$$

Where:

Vol = Realized Volatility

252 = a constant representing the approximate number of trading days in a year

t = a counter representing each trading day

n = number of trading days in the measurement time frame (21 days for a 1VOL and 63 days for a 3VOL)

R_t = continuously compounded daily returns as calculated by the formula:

$$R_t = Ln\frac{P_t}{P_{t-1}}$$

Where:

Ln = natural logarithm

P_t = Underlying Reference Price (closing price) at day *t*

P_{t-1} = Underlying Reference Price at day immediately preceding day *t*

2.1 Volatility Swap under the Heston Model

Under the Heston model, a Volatility swap that has time to maturity *T* can be priced using the following formula:

$$K_{VOL} = \frac{1}{2\sqrt{\pi T}}\int_0^{\infty}\frac{1-Ae^{-\psi v_0 B}}{\psi^{3/2}}d\psi$$

$$A = \left\{\frac{2\phi e^{(\phi+k)T/2}}{(\phi+k)(e^{\phi T}-1)+2\phi}\right\}^{2k\theta/\sigma^2}$$

$$B = \frac{2(e^{\phi T}-1)}{(\phi+k)(e^{\phi T}-1)+2\phi}$$

$$\phi = \sqrt{k^2+2\psi\sigma^2}$$

Where k, θ, v_0, σ are Heston parameters calibrated to the associated option prices.

k is the mean-reverting speed,

θ is the long-term volatility,

v_0 is the initial volatility, and

σ is the volatility of volatility

2.2 Data Selection

Before calibration is performed, it is standard practice to filter the available data set to eliminate outliers and thereby stabilize parameter estimation, ensuring the most efficient calibration. As proposed by Bakshi et al., we remove options that have the following characteristics from the calibration process:

- Options with zero volumes (i.e., non-traded options)
- In-the-money options
- Options with no bid or no ask
- Options with price lower than 0.05

2.3 Calibration

A common solution is to find the Heston parameters that produce the correct market prices of associated options premiums. In other words, although we cannot reconfigure the formula to solve the equation for each parameter, we can furnish an estimate of each value, calculate the result, and then compare the output to the real-world price. If they match, then the estimate was correct. If not, then another estimate is entered, and the whole process starts anew until a match is found. Unfortunately, since we are attempting to estimate four variables at the same time, the process can be quite intense, even for a fast computer.

The most popular approach to solving this problem is to minimize the error or discrepancy between model prices and market prices using the following formula:

$$\min_{\Omega} S(\Omega) = \min_{\Omega} \sum_{i=1}^{N} [C_i^{\Omega}(K_i, T) - C_i^{M}(K_i, T)]^2$$

Where Ω is a vector of parameter values, C^{Ω} and C^{M} are the option prices from the model and market, respectively, with strike K_i and maturity T, and N is the number of options used for calibration.

As for calibration algorithms, we use Differential Evolution (this is a genetic algorithm, which is a global optimizer) and Python "fmin_slsqp" (this is a non-linear least-square algorithm, which is a local optimizer). At the first day of the listed VOLs, we use the Differential Evolution algorithm to calibrate the model. This method gives a global minimum. From the second day on, we use the set of parameters from the previous day as the starting point and use fmin_slsqp to do the calibration. We use this approach since the parameters do not move greatly from day-to-day, so a local optimizer with a good starting point is normally sufficient for our purposes.

2.4 Pricing within the Anticipatory Period

Once we have calculated a theoretical volatility-swap price, we need to adjust that price for the forward-starting feature of a VOL to get the ultimate theoretical value of a RealVol futures contract (TVOL™). At any point during the AP, the TVOL depends on two volatility-swap prices, one expiring at the start date of the CP (the end date of the AP), and the other expiring upon expiration of the VOLs (the end date of the CP). During the AP, the TVOL can be valued by applying an inverse root-mean-square formula to these two theoretical volatility-swap prices.

Let

t_a = time to the end of the AP (which is also the start of the CP),

t_r = time to the end of the CP,

T_1 = time to the front-month option expiration,

T_2 = time to the second-month option expiration,

VOS_{t_a} = volatility swap price expiring at , and

VOS_{t_r} = volatility swap price expiring at . Then,

$TVOL(t, t_a, t_r)$ at time t = the theoretical value of VOL expiring at t_r

We know that

$T_2 = t_r$, since VOL expirations match option expirations, and

$t_a = t_r - 21$, with 21 representing the number of trading days in the CP of a 1VOL.
Similarly, we use 63 in place of 21, representing the number of trading days in the CP of a 3VOL.

The relationship between t_a and T_1 depends on the calendar. Let's assume for now that $T_1 > t_a$.

Suppose that we are at any time $t < t_a$. We select and filter the options that are expiring at T_1 and T_2, then do the calibration based on the methodology outlined for finding a set of Heston parameters. We then calculate VOS_{t_a} and VOS_{t_r} using the volatility-swap formula. Finally, we can calculate the VOL value using the following inverse root-mean-square formula:

$$TVOL(t, t_a, t_r) = \sqrt{\frac{t_r - t}{t_r - t_a} VOS_{t_r}{}^2 - \frac{t_a - t}{t_r - t_a} VOS_{t_a}{}^2}$$

If $T_1 \leq t < t_a$, since the options with maturity T_1 have already expired, we select only those options with T_2 maturity for calibration and pricing.

2.5 Pricing within the Realized-Volatility Period

At any point during the CP, the TVOL depends on both the PVOL and the volatility-swap price with maturity at the end of the CP. The TVOL can be valued by applying a root-mean-square formula to these two quantities.

Let

t_a = time at the start of the CP,

t_r = time to the end of the CP,

T_2 = time to option expiration,

$PVOL(t, t_a)$ = PVOL at time t, and that starts from time t_a, and

VOS_{t_r} = volatility swap price expiring at t_r.

Then,

$TVOL(t, t_a, t_r)$ at time t = the theoretical value of a VOL expiring at t_r.

We know that,

$T_2 = t_r$, since VOL expirations match option expirations, and

$t_a = t_r - 21$, with 21 representing the number of trading days in the CP of a 1VOL, and 63 in place of 21 representing the number of trading days in the CP of a 3VOL.

Suppose that we are at time t ($t_a \le t \le t_r$). We select the options with maturity $T_2 = t_r$, perform the model calibration and price the VOS_{t_r}, and then also calculate the $PVOL(t, t_a)$ based on the VolX daily formula. Finally, the TVOL can be calculated using the following root-mean-square formula:

$$TVOL(t, t_a, t_r) = \sqrt{\frac{t - t_a}{t_r - t_a} PVOL(t, t_a)^2 + \frac{t_r - t}{t_r - t_a} VOS_{t_r}^2}$$

We can see from the formula that when $t = t_a$, the TVOL is determined only by VOS_{t_r}, while when $t = t_r$, it is determined only by $PVOL(t, t_a)$. Thus, upon expiration, the VOL is ultimately settled to the appropriate VolX RVOL index (1RVOL for the 1-month version. or 3RVOL for the 3-month version), which is the realized volatility, as calculated by the VolX daily formula, over the entire period. In other words, the partial volatility (PVOL) converges to realized volatility (RVOL) for the entire CP after all of the data are known.

3. Data

This study covers the period from May 1990 to September 2012 (the start date was determined by the availability of reliable option data). The S&P 500 index, dividend payments, S&P 500 index options, and 3-month T-bill rates are used to calculate the TVOL for 1VOL and 3VOL.

4. Comparison between S&P and S&P with a 1VOL Overlay

The following sections provide comparisons between the performance of an S&P 500 portfolio and a portfolio of the S&P 500 with an IVOL overlay.

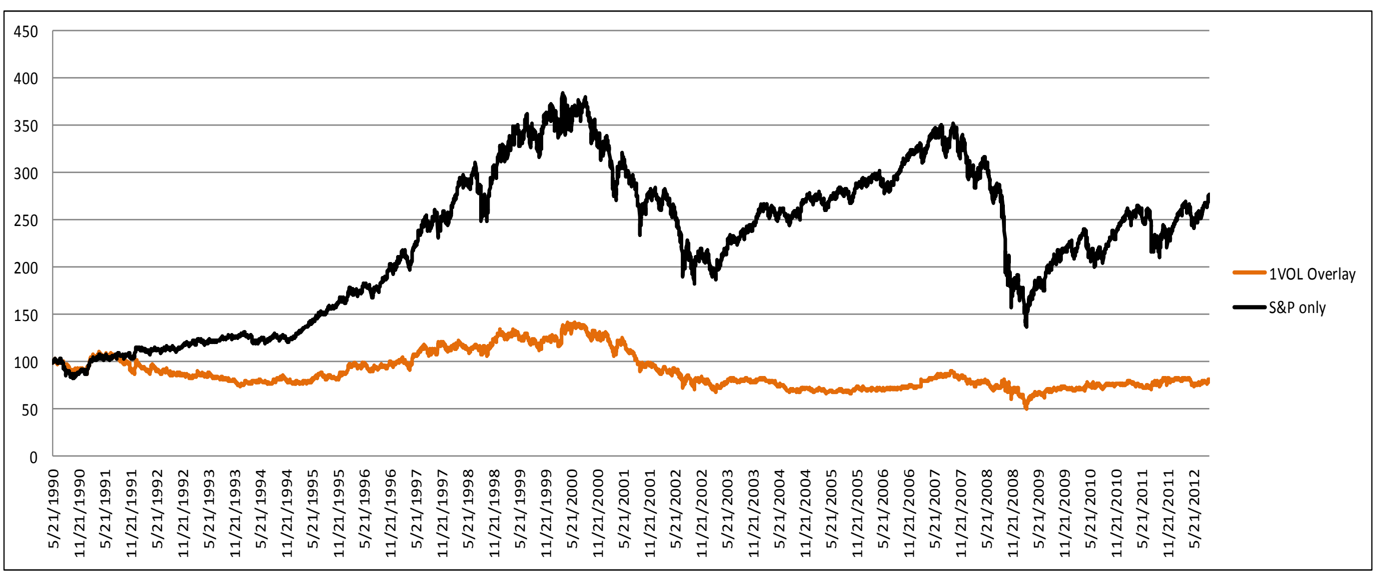

Exhibit 1 Portfolio performance for the whole period 1990-2012
Source: Standard and Poor and Author's calculations

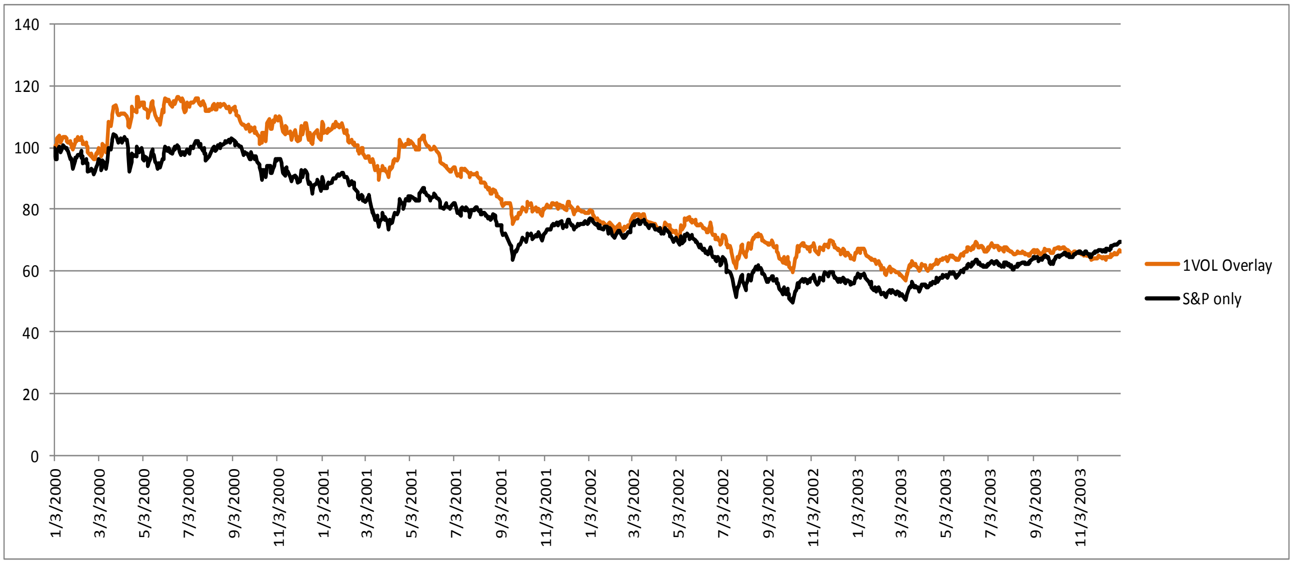

Exhibit 2 Portfolio performance for the period 2000-2003
Source: Standard and Poor and Author's calculations

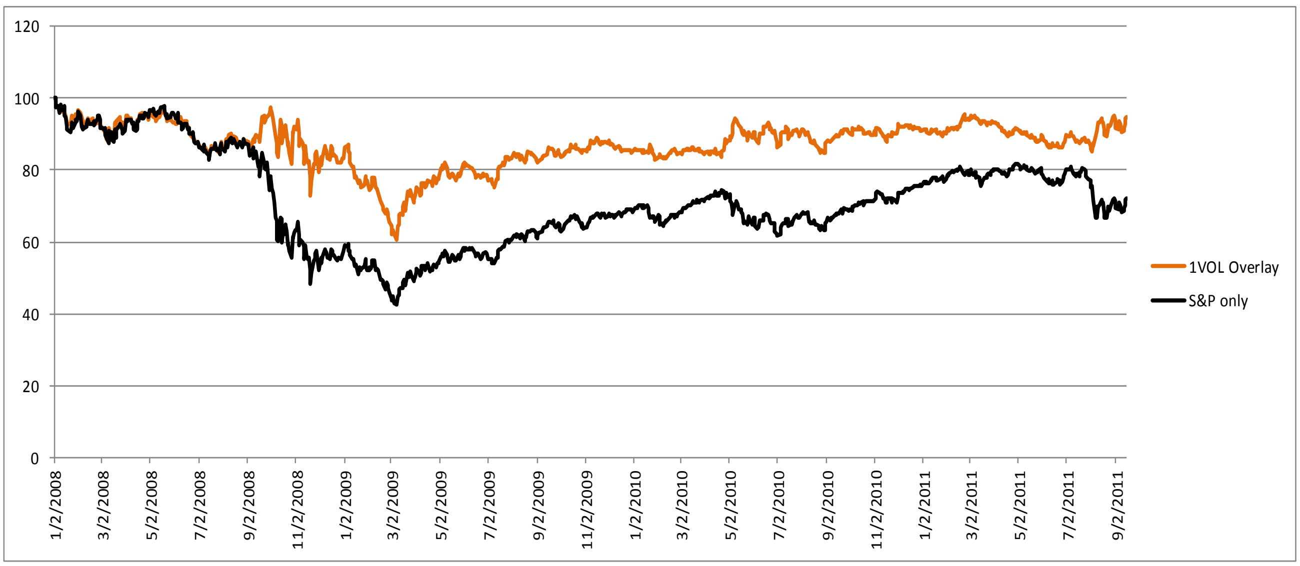

Exhibit 3 Portfolio performance for the period 2008-2011
Source: Standard and Poor and Author's calculations

Exhibit 4 Summary Statistics S&P 500 and B&H 1VOL

Simple buy and hold	1990-2012		2000-2003		2008-2011	
	S&P	B&H 1VOL overlay	S&P	B&H 1VOL overlay	S&P	B&H 1VOL overlay
Annualized Return	4.53%	-1.01%	-9.16%	-10.34%	-8.86%	-1.43%
Annualized Standard Deviation	18.75%	17.91%	21.93%	20.33%	28.84%	23.25%

Source: Standard and Poor and Author's calculations

4.1 A simple buy-and-hold strategy

Studies have shown that pure long volatility exposure generally results in negative returns over the long term. Our research led to similar findings. The B&H 1VOL overlay portfolio has a buy-and-hold or continual 10% allocation to 1-month RealVol futures along with a 100% allocation to the S&P total-return index. The 10% VOL allocation is established at the start of the period and is held until the VOL expires. The 10% VOL allocation is then reestablished the following month and the process repeats for each month in the study period.

We compare the performance of the S&P portfolio to the simple B&H 1VOL overlay portfolio in three periods. 1990–2012 (all data), 2000–2003 (dot-com collapse), and 2008–2011 (credit crisis) are illustrated in Exhibits 1, 2, and 3, respectively. We can observe that, under the simple buy-and-hold strategy, the B&H 1VOL overlay performs poorly over the whole period. While the overlay indeed reduced risk, the drag on performance over the whole period was so great that a long-term strategy of buying volatility on a regular basis is not expected to generate positive returns. However, for the market break in the 2000-2003 period, the B&H 1VOL overlay beat the S&P portfolio most of the time except when the market resumed its rise. For the 2008-2011 period, the B&H 1VOL overlay performed well, exhibiting a higher return than the S&P portfolio.

Exhibit 4 provides summary statistics for this B&H 1VOL strategy over the three time periods.

We can see that this simple B&H 1VOL overlay strategy helped reduce the realized volatility of an S&P portfolio, but the allocation to VOLs needed to be more judicious in order to improve returns.

Exhibit 5 Performance of S&P 500 252MA, 63MA, and 21MA 1VOL = 2008-2011

1990–2012	S&P	252MA 1VOL overlay	63MA 1VOL overlay	21MA 1VOL overlay
Annualized Return	4.53%	4.67%	4.04%	2.90%
Annualized Standard Deviation	18.75%	17.64%	17.51%	17.81%

Source: Standard and Poor and Author's calculations

4.2 A Simple Active Moving-Average Strategy

Since the B&H 1VOL overlay did not generally outperform the S&P portfolio in terms of returns, we searched for some simple, active VOL overlay strategies that might have proven useful for long-term investors. We tested a moving-average indicator over three measurement periods: (1) short-term, 21-day (one month, 21MA), (2) medium-term, 63-day (three months, 63MA), and (3) long-term, 252-day (12 months, 252MA). Each indicator was tested separately. The concept of the strategy is simple: Upon expiration of each VOL, we look to the moving-average indicator to decide on the allocation for the next-to-expire VOL. When the moving-average indicator

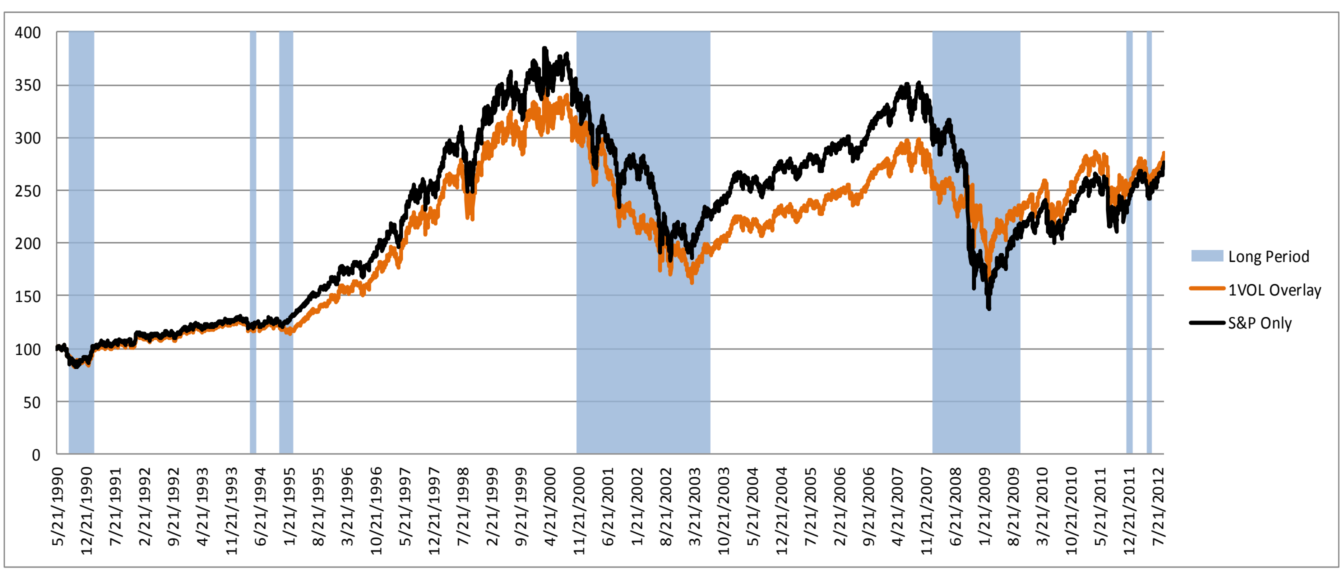

Exhibit 6 Portfolio performance for the whole period 1990-2012 under 252MA 1VOL
Source: Standard and Poor and Author's calculations

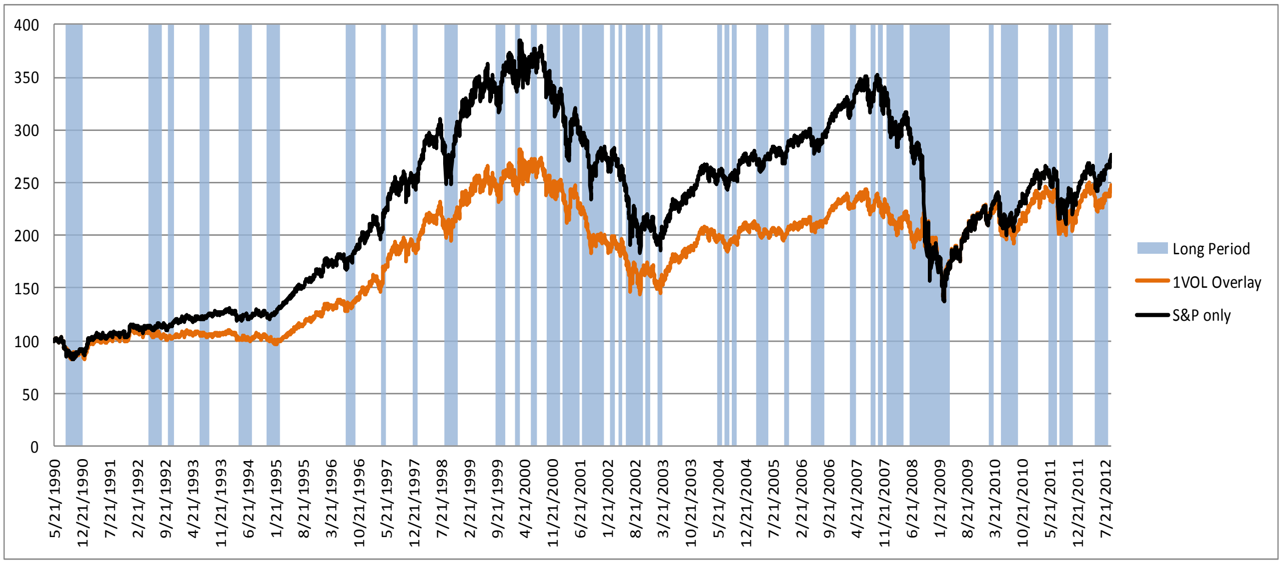

Exhibit 7 Portfolio performance for the whole period 1990-2012 under 63MA 1VOL
Source: Standard and Poor and Author's calculations

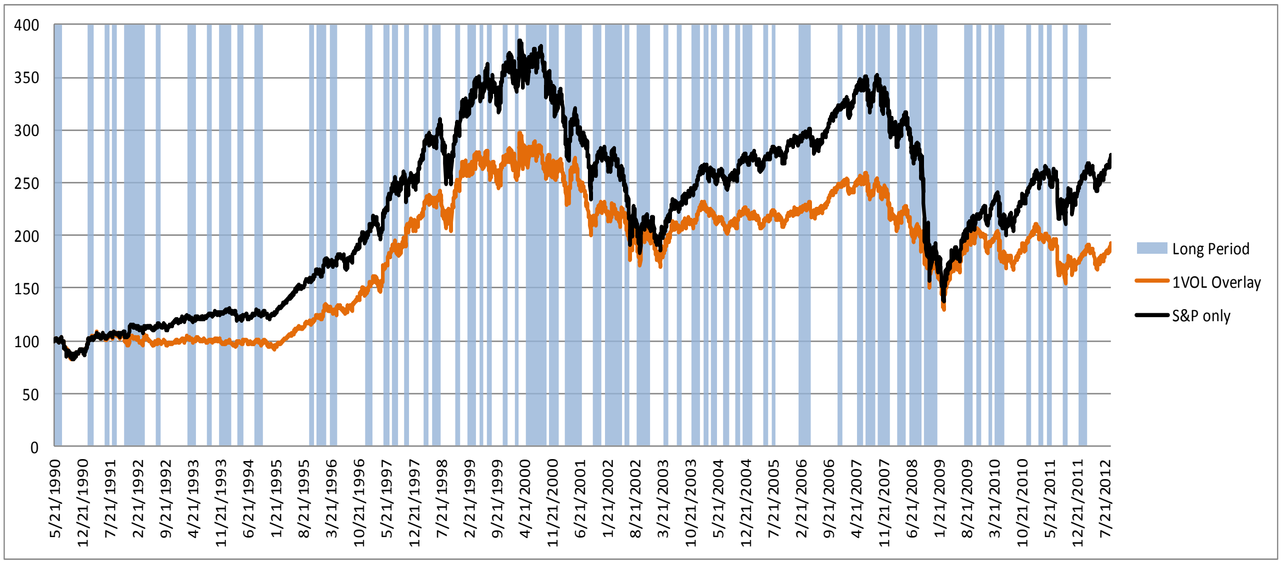

Exhibit 8 Portfolio performance for the whole period 1990-2012 under 21MA 1VOL
Source: Standard and Poor and Author's calculations

Exhibit 9 Performance of S&P 500 252MA, 63MA, and 21MA 1VOL - 2000–2003

2000–2003	S&P	252MA 1VOL overlay	63MA 1VOL overlay	21MA 1VOL overlay
Annualized Return	–9.16%	–10.61%	–7.19%	–5.78%
Annualized Standard Deviation	21.93%	20.71%	20.99%	21.16%

Source: Standard and Poor and Author's calculations

is rising, we allocate 0% of the portfolio value to buying VOLs; when the moving-average indicator is falling, we allocate 10% to buying VOLs.

Exhibits 5, 6, 7 and 8 show the performance of the 252MA, 63MA, and 21MA 1VOL overlay portfolios. The blue shaded bars indicate times in which the portfolios had a long exposure in 1VOLs. We can see that all three strategies achieved performance that was considerably better than the B&H 1VOL overlay approach. In fact, the realized volatility of each overlay portfolio was again lower than the S&P; however, the overlay portfolios all

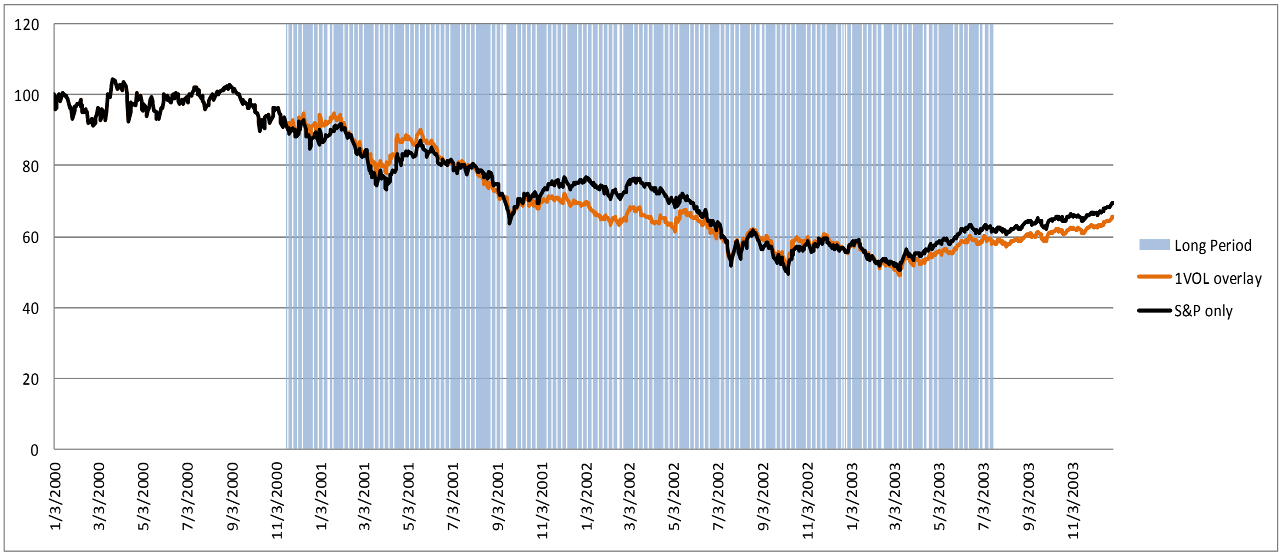

Exhibit 10 Portfolio performance for the sub-period 2000-2003 under 252MA
Source: Standard and Poor and Author's calculations

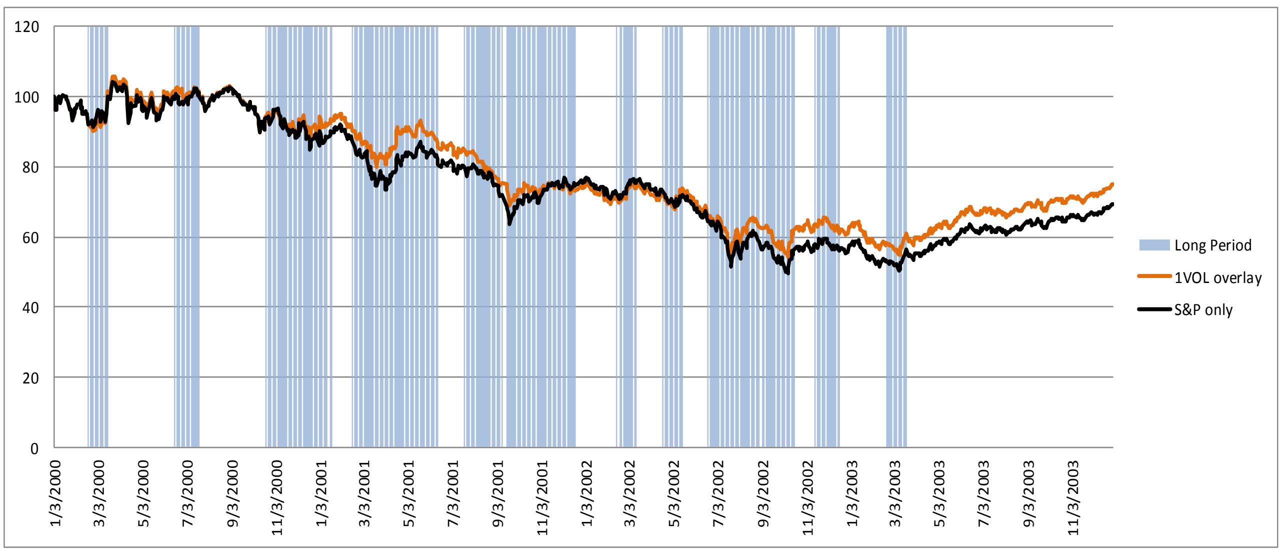

Exhibit 11 Portfolio performance for the sub-period 2000-2003 under 63MA
Source: Standard and Poor and Author's calculations

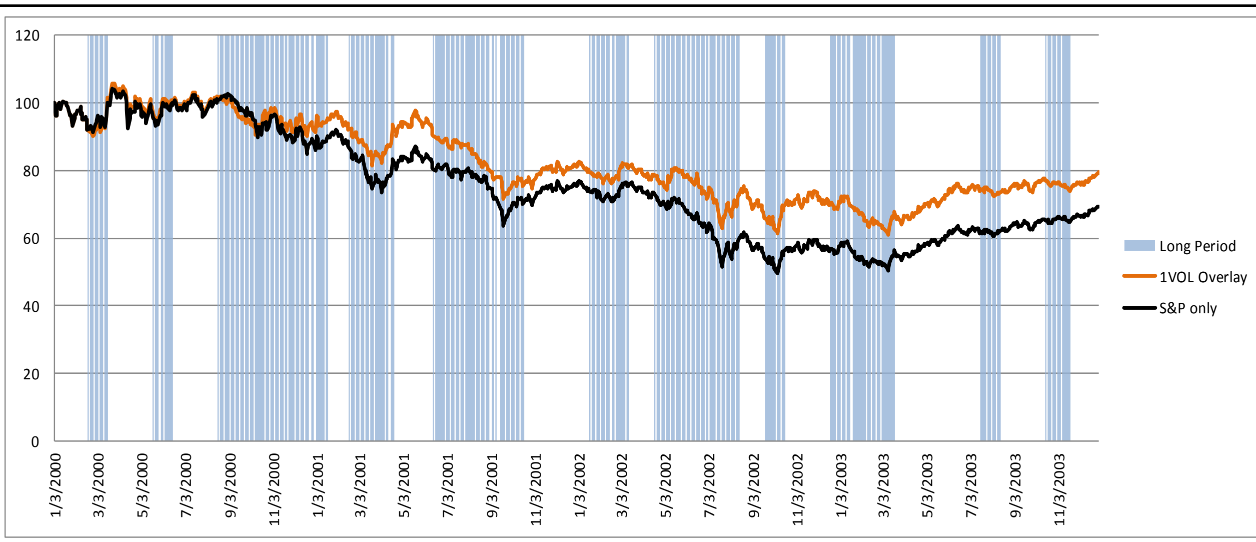

Exhibit 12 Portfolio performance for the sub-period 2000-2003 under 21MA
Source: Standard and Poor and Author's calculations

Exhibit 13 Performance of S&P 500 252MA, 63MA, and 21MA 1VOL - 2008–2011

2008–2011	S&P	252MA 1VOL overlay	63MA 1VOL overlay	21MA 1VOL overlay
Annualized Return	–8.86%	–2.29%	–1.66%	–8.14%
Annualized Standard Deviation	28.84%	25.14%	25.15%	25.60%

Source: Standard and Poor and Author's calculations

exhibited returns lower than the S&P portfolio, with the exception of the 252MA 1VOL overlay portfolio, which had a slightly higher return than the S&P portfolio.

Next, we focus on two down-market periods. Exhibits 9, 10, 11, and 12 provide the performance of the 1VOL overlay portfolios under three MA strategies during the dot-com bubble collapse of 2000–2003. We can see that all the 1VOL overlay portfolios lowered the standard deviation of the S&P portfolio by approximately one percentage point. Furthermore, 63MA 1VOL overlay increased the annual return from –9.16% to –7.19%, and 21MA 1VOL overlay from –9.16% to –5.78% during this market downturn. The 21MA 1VOL overlay strategy provides the highest return in this period.

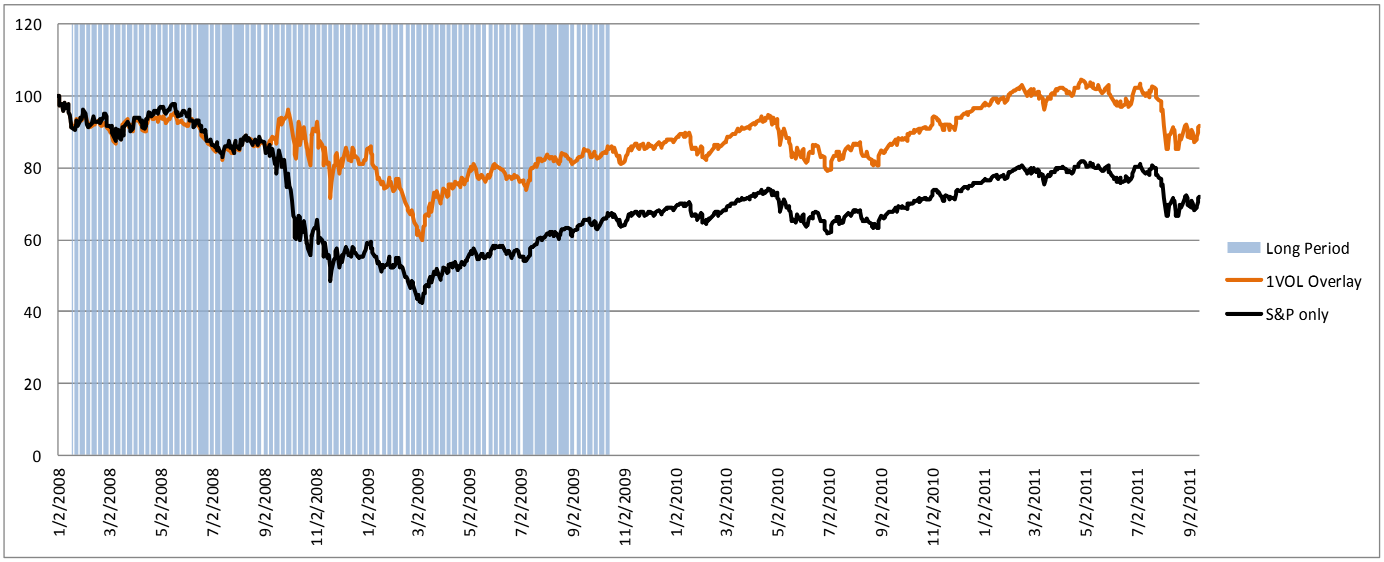

Exhibit 14 Portfolio performance for the sub-period 2008-2011 under 252MA 1VOL
Source: Standard and Poor and Author's calculations

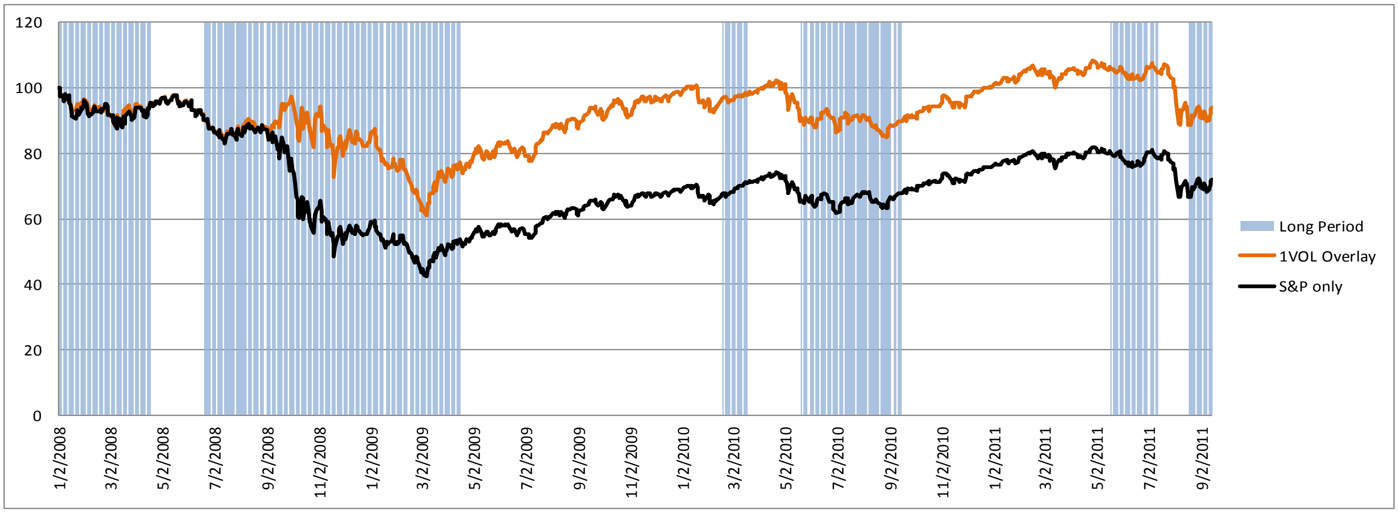

Exhibit 15 Portfolio performance for the sub-period 2008-2011 under 63MA 1VOL
Source: Standard and Poor and Author's calculations

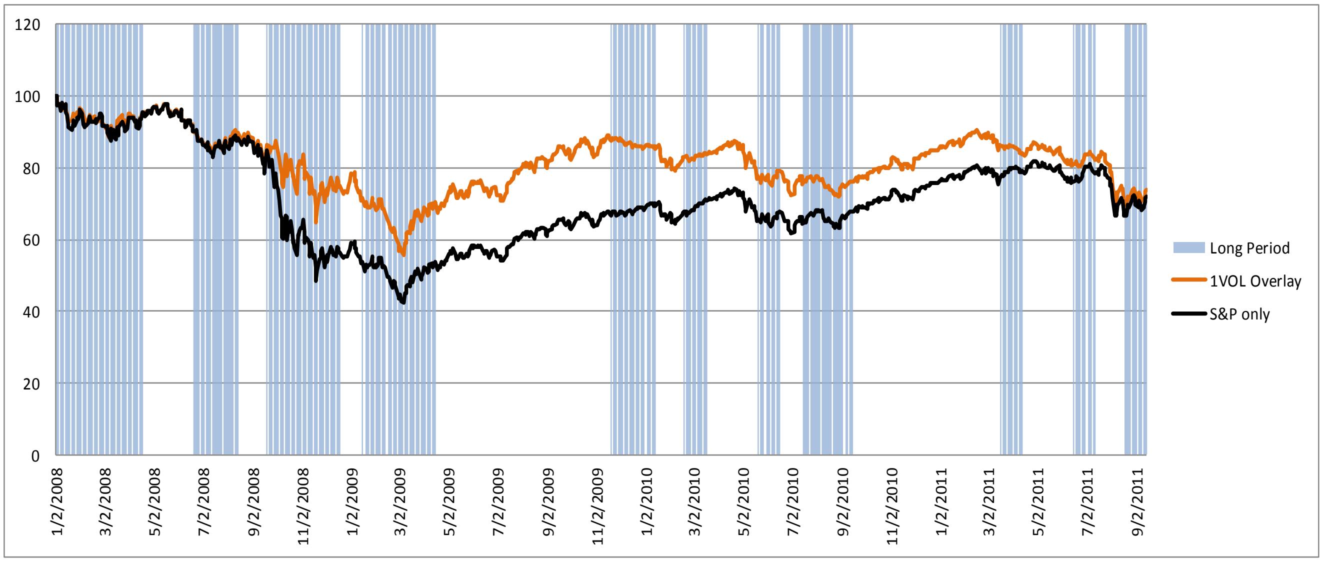

Exhibit 16 Portfolio performance for the sub period 2008-2011 under 21MA 1VOL
Source: Standard and Poor and Author's calculations

Next, we focus on the market break during the credit crisis. Exhibits 13, 14, 15, and 16 illustrate the performance of the portfolios for 2008–2011. In this case, the 1VOL overlays lowered the standard deviation by approximately three percentage points on average and at the same time increased the return. 63MA 1VOL overlay provided the highest return during this period.

From the results, we can conclude that both a simple buy-and-hold and an active moving-average strategy, using 1VOLs as an overlay, helped reduce the standard deviation of an S&P portfolio. While a buy-and-hold strategy yielded negative returns over time, a simple active 1VOL overlay, using moving averages for the signal, provided protection and increased returns on an S&P portfolio during market downturns.

5. A Slightly More Complex Strategy

The following sections provide comparisons of various investment strategies involving VOLs.

5.1 Long Only 252/21MA Strategy

Many academic papers discuss how the inverse correlation between realized volatility and rising markets is not as strong as when markets are falling. Therefore, in order to capture this bifurcation of volatility and its relationship to equity price movement, we consider a strategy that is slightly more complex. We do so by using the slow, or long-term, 252MA on the upside and the fast, or short-term, 21MA for the downside. In other words, we have three states: When the 252MA is rising, we allocate 0%; when the 252MA is falling, we then look to the 21MA. If the 21MA is rising, we allocate 0%. If the 21MA is also falling, we allocate 10%. We call this the 252/21MA 1VOL overlay portfolio. In this section of the study, we only consider buying the 1VOLs.

Exhibits 17, 18, 19, and 20 summarize the performance of 1VOL overlay portfolios under the 252/21MA strategy, compared to all other strategies. It is clear that the 252/21MA strategy provided superior performance. It both reduced standard deviation and increased returns. The 252/21MA strategy generated higher returns than the S&P portfolio as well as all three moving-average portfolios for 1990–2012 and 2000–2003, while not appreciably increasing (and sometimes decreasing) standard deviation.

Exhibit 17 Performance of S&P 500 252MA, 63MA, 21MA, and 252/21MA 1VOL - 1990–2011 and Sub-Periods

1990-2012	S&P	252MA 2VOL overlay	63MA 1VOL overlay	21MA 1VOL overlay	252/21MA 1VOL overlay
Annualized Return	4.53%	4.67%	4.04%	2.90%	6.37%
Annualized Standard Deviation	18.75%	17.64%	17.51%	17.81%	17.25%
2000-2003	S&P	252MA 1VOL overlay	63MA 1VOL overlay	21MA 1VOL overlay	252/21MA 1VOL overlay
Annualized Return	-9.16%	-10.61%	-7.19%	-5.78%	-5.10%
Annualized Standard Deviation	21.93%	20.71%	20/99%	21.16%	20.99%
2008-2011	S&P	252MA 1VOL overlay	63MA 1VOL overlay	21MA 1VOL overlay	252/21 MA 1VOL overlay
Annualized Return	-8.86%	-2.29%	-1.66%	-8.14%	-1.96%
Annualized Standard Deviation	28.84%	25.14%	25.15%	25.60%	26.10%

Source: Standard and Poor and Author's calculations

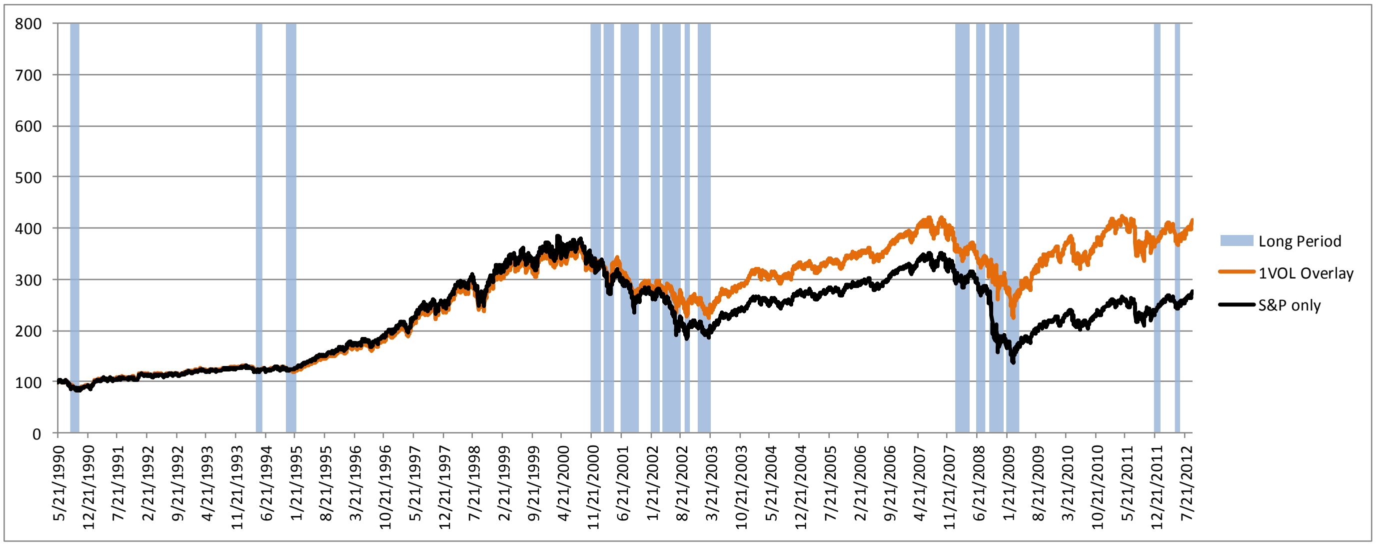

Exhibit 18 Portfolio performance for the whole period 1990-2012 under 252/21MA 1VOL
Source: Standard and Poor and Author's calculations

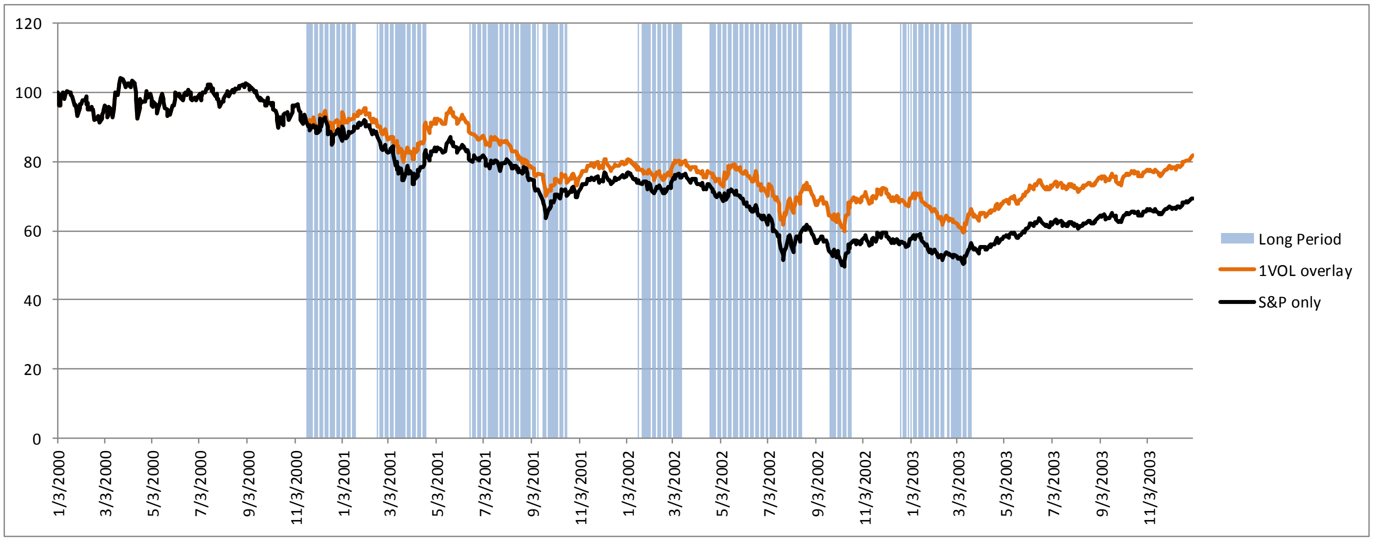

Exhibit 19 Portfolio performance for the sub-period 2000-2003 under 252/21MA 1VOL
Source: Standard and Poor and Author's calculations

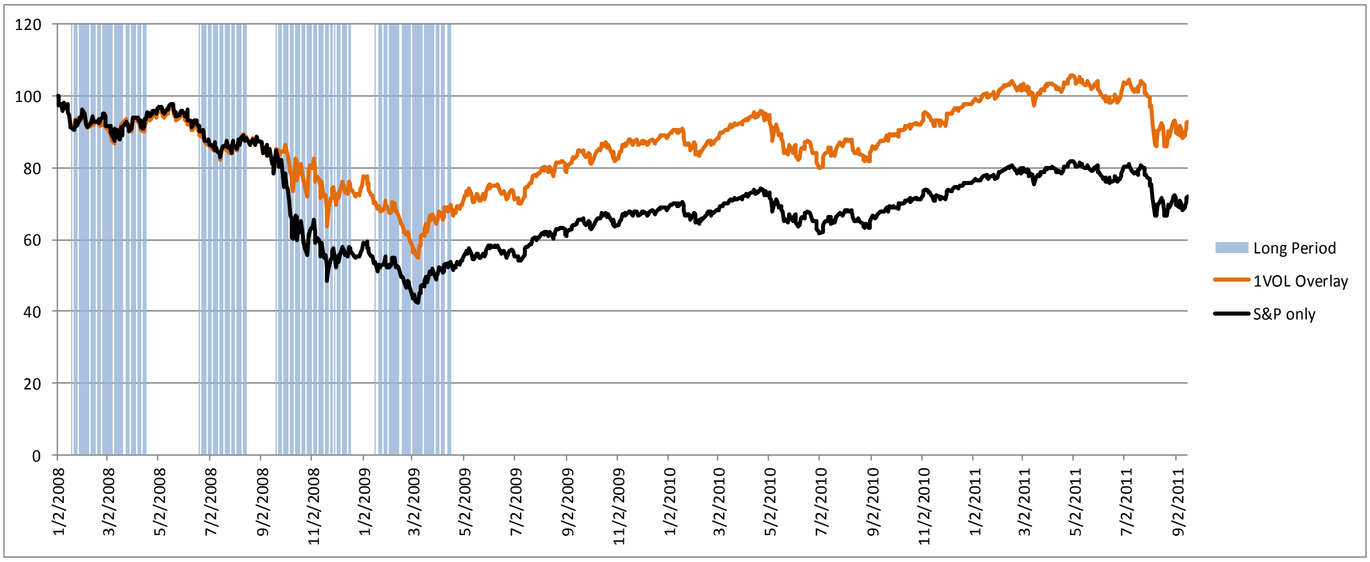

Exhibit 20 Portfolio performance for the sub-period 2008-2011under 252/21MA 1VOL
Source: Standard and Poor and Author's calculations

5.2 Consider Selling the VOLs

So far, we have only considered buying VOLs; we now discuss how an active short position might affect the performance of the overlay portfolio. When the 252MA is rising, we allocate –2% (in other words, we sell the 1VOL at 2% of the portfolio value); when the 252MA is falling, we then look to the 21MA. If the 21MA is rising, we allocate 0% (just as before), and if the 21MA is falling, we allocate 10% (just as before). Exhibits 21, 22, 23, and 24 illustrate the performance of the 252/21MA 1VOL overlays when short positions are permitted.

Over the 1990 to 2012 period, the exhibits indicate that having an active short and long 1VOL overlay, using our more complex but still simple to implement 252/21MA 1VOL overlay strategy, further increased the return of the B&H 1VOL overlay portfolio, but at the cost of a slightly higher standard deviation. Adding a 2% short position in the 1VOL overlay increased portfolio returns from 6.37% to 7.30% in the periods 1990-2012, but slightly decreased returns in the periods 2000-2003 and 2008-2011, with standard deviations still slightly lower than those of the S&P portfolio.

Exhibit 21 Performance of S&P 500 and 252/21MA 1VOL with and without Short Positions - 1990–2011 and Sub-Periods

1990–2012	S&P	252/21MA 1VOL overlay	252/21MA 1VOL overlay (-2%, 0%, 10%)
Annualized Return	4.53%	6.37%	7.30%
Annualized Standard Deviation	18.75%	17.25%	18.45%
2000–2003			
Annualized Return	–9.16%	–5.10%	–5.18%
Annualized Standard Deviation	21.93%	20.99%	21.36%
2008–2011			
Annualized Return	–8.86%	–1.96%	–2.39%
Annualized Standard Deviation	28.84%	26.10%	27.11%

Source: Standard and Poor and Author's calculations

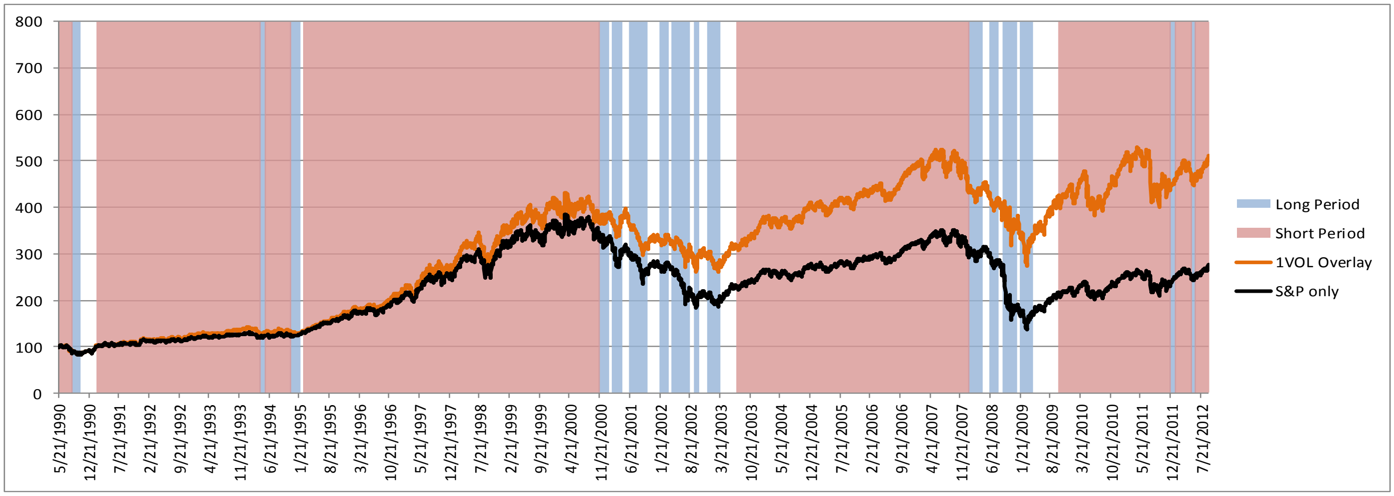

Exhibit 22 Portfolio performance for the whole period 1990-2012 under 252/21MA 1VOL overlay, with –2%, 0%, 10% allocation
Source: Standard and Poor and Author's calculations

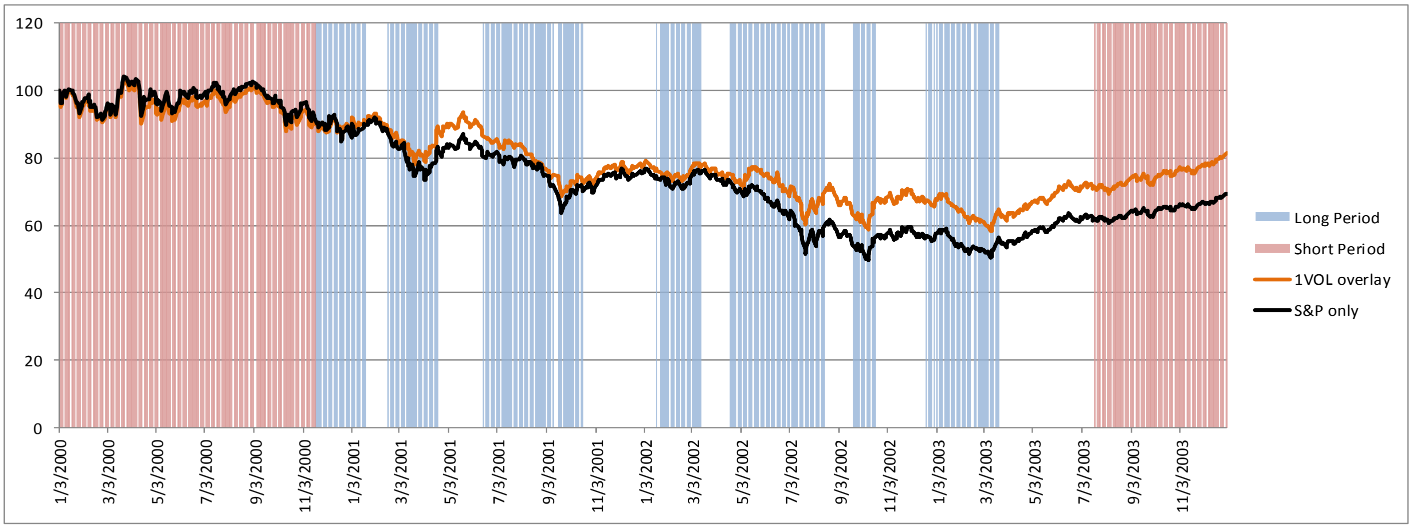

Exhibit 23 Portfolio performance for the sub-period 2000-2003 under 252/21MA 1VOL overlay, with –2%, 0%, 10% allocation
Source: Standard and Poor and Author's calculations

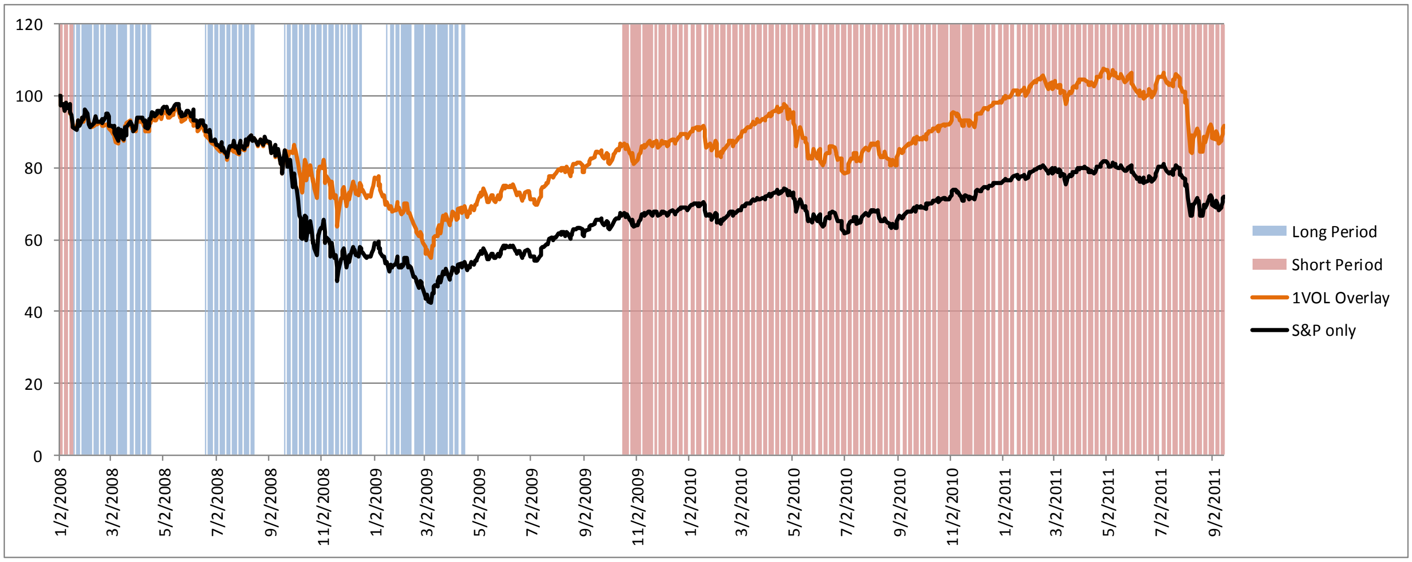

Exhibit 24 Portfolio performance for the sub-period 2002-2011 under 252/21MA 1VOL overlay, with –2%, 0%, 10% allocation
Source: Standard and Poor and Author's calculations

As a final strategy using 1VOLs, we consider a different allocation under the same 252/21MA approach. This time, we allocate –2% when 252MA is rising (just as before), and when the 252MA is falling, we look to the 21MA. When the 21MA is rising, we allocate 0% (just as before) but now allocate 20% when 21MA is falling. We doubled the long volatility hedge to see if it could better protect the portfolio during times of economic stress, with the hope of increasing the overall returns. Exhibit 25 and 26 provide the performance of the 1VOL overlay portfolios with such allocation over the same three time frames. However, we show only the chart for the whole period. It can be observed from the exhibits that adding an additional 10% allocation when 21MA is falling further enhances the return of the 1VOL overlay portfolio, while standard deviations remain lower than the S&P portfolio for the period 2008-2011.

6. Comparison between 1VOL and 3VOL Overlay on an S&P Portfolio

So far, we have been studying only strategies employing 1VOLs. We now provide a comparison between the 1VOL and 3VOL overlay portfolios. Exhibits 27, 28, 29, and 30 show the performance of 3VOL overlay portfolios for the whole period under the 252/21MA strategies, with 0%, 0%, 10%; –2%, 0%, 10%; and –2%, 0%, 20% allocations, respectively. The exhibits indicate that both 1VOL and 3VOL overlays under the 252/21MA strategy can help

Exhibit 25 Performance of S&P 500 and 252/21MA 1VOL with Varied Allocation Levels - 1990–2011 and Sub-Periods

1990–2012	S&P	252/21MA 1VOL overlay (–2%, 0%, 10%)	252/21MA 1VOL overlay (–2%, 0%, 20%)
Annualized Return	4.53%	7.30%	8.78%
Annualized Standard Deviation	18.75%	18.45%	18.76%
2000–2003			
Annualized Return	–9.16%	–5.18%	–1.12%
Annualized Standard Deviation	21.93%	21.36%	23/61%
2008–2011			
Annualized Return	–8.86%	–2.39%	3.44%
Annualized Standard Deviation	28.84%	27.11%	27.71%

Source: Standard and Poor and Author's calculations

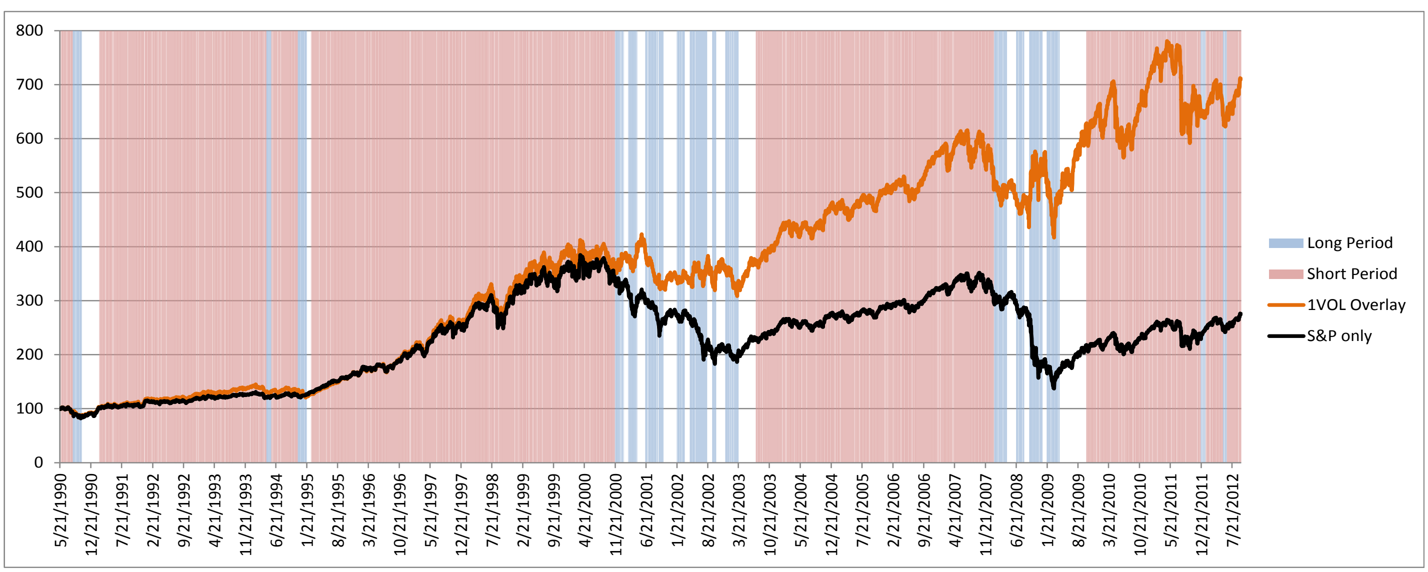

Exhibit 26 Portfolio performance for the whole period 1990-2012 under 252/21MA 1VOL overlay, with –2%, 0%, 20% allocation

Source: Standard and Poor and Author's calculations

Exhibit 27 Performance of 252/21MA 1VOL and 3VOL - 1990–2011 and Sub-Periods

1990–2012 (0%, 0%, 10%)	252/21MA 1VOL overlay	252/21MA 3VOL overlay
Annualized Return	6.37%	5.15%
Annualized Standard Deviation	17.25%	17.72%
1990–2012 (–2%, 0%, 10%)		
Annualized Return	7.30%	5.60%
Annualized Standard Deviation	18.45%	18.31%
1990–2012 (–2%, 0%, 20%)		
Annualized Return	8.78%	6.08%
Annualized Standard Deviation	18.76%	18.00%

Source: Standard and Poor and Author's calculations

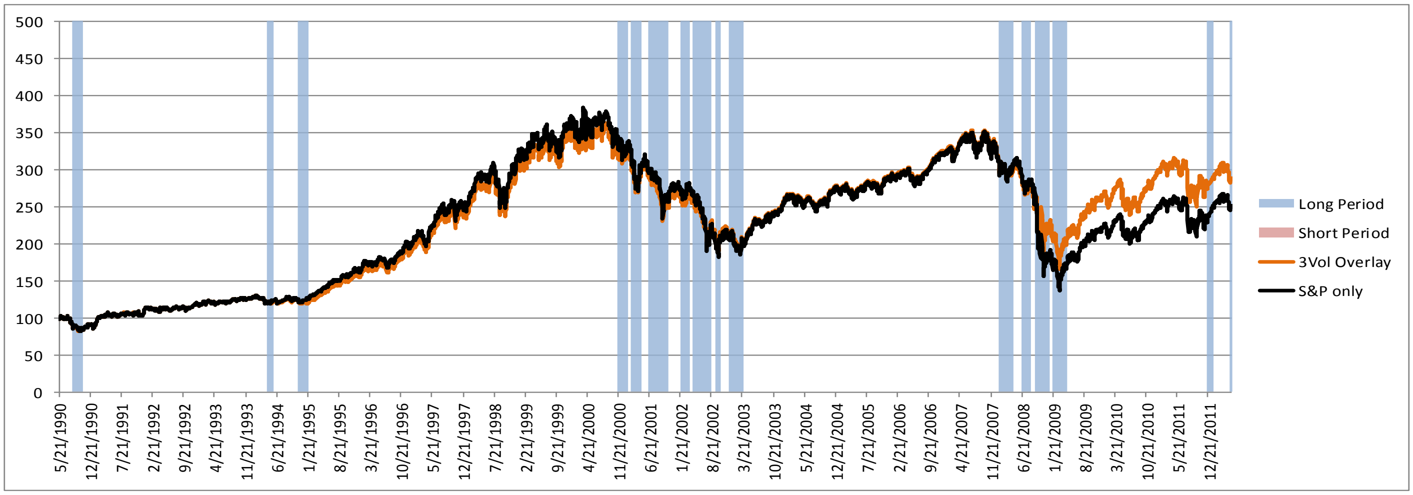

Exhibit 28 Portfolio performance for the whole period 1990-2012 under 252/21MA, 3VOL overlay, with 0%, 0%, 10% allocation
Source: Standard and Poor and Author's calculations

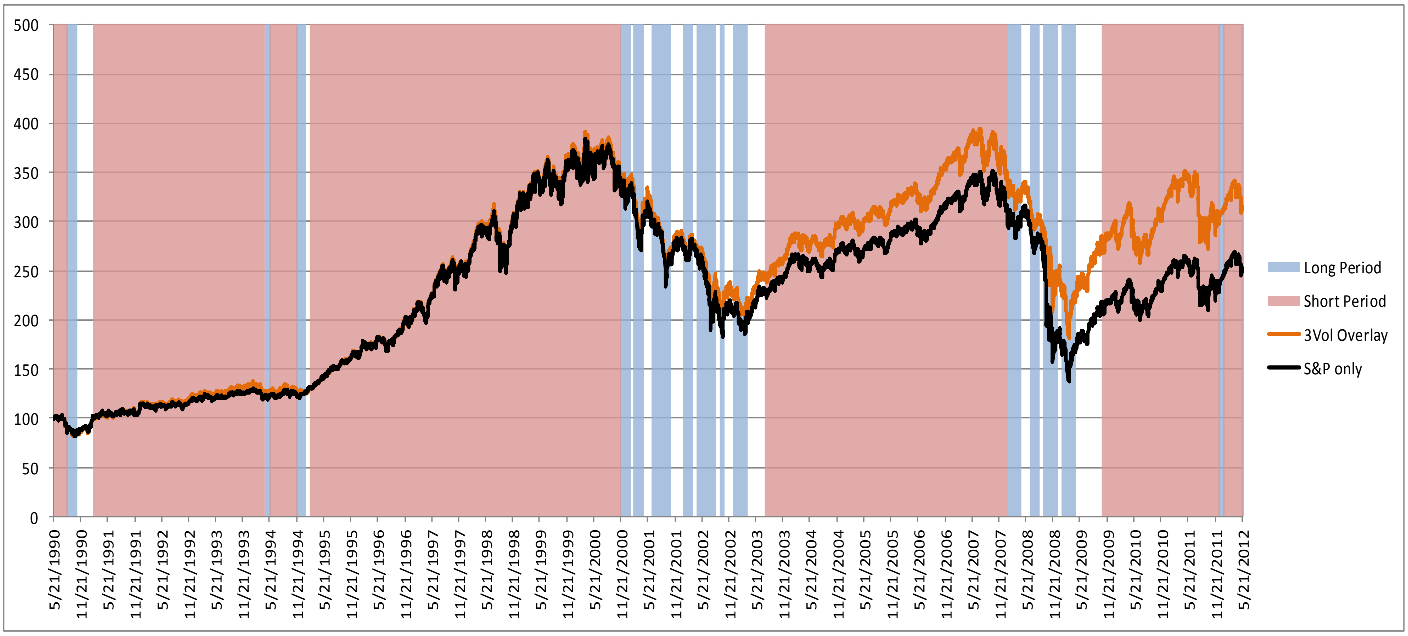

Exhibit 29 Portfolio performance for the whole period 1990-2012 under 252/21MA, 3VOL overlay, with –2%, 0%, 10% allocation
Source: Standard and Poor and Author's calculations

increase returns and reduce risk. Also, for –2%, 0%, 10% and –2%, 0%, 20% allocations, 1VOL has a higher return than the 3VOL overlay, while 3VOL has a smaller standard deviation than the 1VOL overlay. Finally, for the long-only overlays, the 1VOL overlay had a higher return and a smaller standard deviation than the 3VOL overlay.

7. Conclusion

Exhibits 31 and 32 summarize the results of the paper.

Our research indicates that the 1-month RealVol futures (1VOL) overlay portfolio under a simple buy-and-hold (B&H) strategy would have resulted in lower standard deviation to the simple buy-and-hold equity portfolio (S&P). However, the cost to reduce that risk was substantially reduced returns. We postulated that if one could devise a simple active allocation approach that added RealVol futures exposure during market breaks and eliminated exposure during market rises, the portfolio's performance could be enhanced. We proposed three simple moving-average indicators (21-day moving average, 21MA; 63-day moving average, 63MA; and 252-day moving average, 252MA) and one slightly more complex indicator that combined the long-term with the short-term (252- and 21-day moving average, 252/21MA). When we did so, the portfolio returns increased and the standard deviation decreased.

We used a simple active moving-average strategy to decide when to allocate capital to the 1-month RealVol futures. Returns from this strategy are better than the simple buy-and-hold strategy. The 1-month RealVol futures overlay portfolio under this simple moving-average strategy outperforms the total-return S&P portfolio during market downturns. The 1-month RealVol futures overlay, regardless of which moving-average timeframe we followed, resulted in higher returns and reduced standard deviations during market breaks. Such results significantly helped the portfolio weather the economic storms. However, when considering the performance during both bullish and bearish markets over 23 years, the results were still not ideal.

Therefore, we introduced one slightly more complex strategy: The 252/21MA 1VOL overlay strategy. Performance under this strategy improved significantly. The 252/21MA 1VOL overlay beat the total-return S&P portfolio both in the entire period and in the sub-periods. Next, we allowed a small active short position in RealVols only when the long-term moving average was rising. The return of the 252/21MA 1VOL overlay portfolio with an active short position further increased returns but at the expense of a slightly higher standard deviation for the entire period, but decreased returns for the sub periods, indicating that it would not be a good idea to short volatility during market crashes.

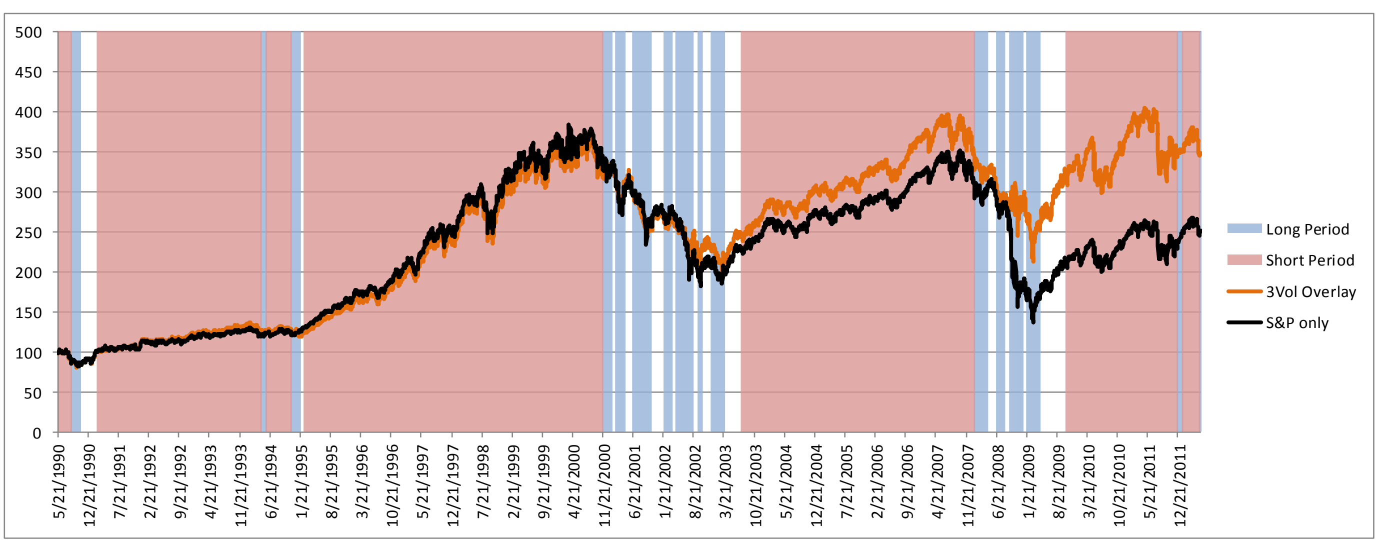

Exhibit 30 Portfolio performance for the whole period 1990-2012 under 252/21MA, 3VOL overlay, with –2%, 0%, 20% allocation

Source: Standard and Poor and Author's calculations

Finally, we compared the performance of the 1VOL to the 3-month RealVol futures (3VOL) overlay portfolio, under the same 252/21MA strategy. Our results showed that the 1VOL overlay had a higher return with greater risk, than the 3VOL overlay.

Although we have shown that the RealVol futures overlay portfolios under the 252/21MA strategy reduced risk and increased returns, more research is needed. For one, RealVol futures have not started trading. The lack of market prices required us to use pricing models to determine theoretical prices. Had RealVol futures been trading in the past, there is no guarantee that investors could have executed trades at such theoretical prices. In addition, even if such pricing were available, there is no guarantee that the results we have shown are representative of the future. Markets can and will sometimes exhibit non-theoretical pricing behavior. Also, the results have not taken into account the costs of actually trading a market with bid/ask spreads and commissions. There may be other factors that could make our results differ from reality. Readers may want to experiment with other indicators and/or more sophisticated decision methodologies and strategies. In any event, it appears that judicious use of RealVol futures in conjunction with an equity portfolio may be a welcome addition to an investor's arsenal of exchange-traded instruments.

Exhibit 31 Summary of 1VOL Results

1990–2012	S&P	1 VOL Overlay						
		B&H	252MA	63MA	21MA	252/21MA (0%, 0%, 10%)	252/21MA (–2%, 0%, 10%)	252/21MA (–2%, 0%, 20%)
Annualized Return	4.53%	–1.01%	4.67%	4.04%	2.90%	6.37%	7.30%	8.78%
Annualized S.D.	18.75%	17.91%	17.64%	17.51%	17.81%	17.25%	18.45%	18.76%
2000–2003								
Annualized Return	–9.16%	–10.34%	–10.61%	–7.19%	–5.78%	–5.10%	–5.18%	–1.12%
Annualized S.D.	21.93%	20.33%	20.71%	20.99%	21.16%	20.99%	21.36%	23.61%
2008–2011								
Annualized Return	–8.86%	–1.43%	–2.29%	–1.66%	–8.14%	–1.96%	–2.39%	3.44%
Annualized S.D.	28.84%	23.25%	25.14%	25.15%	25.60%	26.10%	27.11%	27.71%

Source: Standard and Poor and Author's calculations

Exhibit 32 Summary of 3VOL Results

1990–2012	S&P	3VOL Overlay			
		B&H	252/21MA (0%, 0%, 10%)	252/21MA (–2%, 0%, 10%)	252/21MA (–2%, 0%, 20%)
Annualized Return	4.53%	1.64%	5.15%	5.60%	6.08%
Annualized S.D.	18.75%	15.89%	17.72%	18.31%	18.00%
2000–2003					
Annualized Return	–9.16%	–11.66%	–7.90%	–7.74%	–6.25%
Annualized S.D.	21.93%	19.09%	20.40%	20.64%	20.17%
2008–2011					
Annualized Return	–8.86%	–4.41%	–4.29%	–4.50%	–0.89%
Annualized S.D.	28.84%	22.95%	26.16%	26.85%	25.85%

Source: Standard and Poor and Author's calculations

Please note: VolX has agreed to make some of these data available on its web site. Go to volx.us and click on the menus Data \ VolX Products Research.

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VolX daily formula, http://www.volx.us/VolFormula.htm

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